Algebra Seminar

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Fine gradings of exceptional simple Lie algebras

Abstract:

In the past two decades, there has been much progress in the study of gradings on simple Lie algebras by arbitrary groups. In particular, over an algebraically closed field of characteristic zero, fine gradings have been classified for all finite-dimensional simple Lie algebras except E7 and E8.

In a recent paper, A. Elduque has shown that, in a sense, a fine grading splits into two independent gradings: a grading by a free abelian group, which is also a grading by a root system, and a fine grading by a finite group on the corresponding coordinate algebra. For example, the Z_2^3-grading on the algebra of octonions that arises from the three iterations of Cayley-Dickson doubling process is "responsible" not only for a fine Z_2^3-grading on G2 but also for fine gradings on F4 by the group Z times Z_2^3 and on E6, E7 and E8, respectively, by Z^2 times Z_2^3, Z^3 times Z_2^3 and Z^4 times Z_2^3. We have a similar picture for the Z_3^3-grading on the simple exceptional Jordan algebra that can be obtained from the first Tits construction.

In this talk, we will discuss the above Z_2^3- and Z_3^3-gradings as well as the Z_4^3-grading on the simple structurable algebra of dimension 56, the double of the Jordan algebra of Hermitian matrices of order 4 over quaternions. Together, these three gradings are "responsible" for almost all known fine gradings on exceptional simple Lie algebras.