

Some Problems about Logarithms for Enrichment¹

1. Determine all values of x for which $\log_5(x - 2) + \log_5(x - 6) = 1$.
2. If $a^2 + b^2 = 7ab$, prove that $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$.
3. If $\sqrt{2^x} - \frac{12}{\sqrt{2^x}} = 1$, find x .
4. Solve $\log_2 2x = \log_4 x$.
5. The points $A(x_1, y_1)$ and $B(x_2, y_2)$ lie on the graph of $y = \log x$. Through the midpoint D of the line segment AB , a horizontal line is drawn and this cuts the graph at $C(x_3, y_3)$. Prove that $x_3^2 = x_1x_2$.

¹borrowed from some books of Canadian Mathematics Competition Problems produced by the University of Waterloo