Some Problems about Logarithms for Enrichment¹

- 1. Determine all values of x for which $\log_5(x-2) + \log_5(x-6) = 1$.
- 2. If $a^2 + b^2 = 7ab$, prove that $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$.
- 3. If $\sqrt{2^x} \frac{12}{\sqrt{2^x}} = 1$, find *x*.
- 4. Solve $\log_2 2x = \log_4 x$.
- 5. The points $A(x_1, y_1)$ and $B(x_2, y_2)$ lie on the graph of $y = \log x$. Through the midpoint D of the line segment AB, a horizontal line is drawn and this cuts the graph at $C(x_3, y_3)$. Prove that $x_3^2 = x_1 x_2$.

 $^{^{1}\}mathrm{borrowed}$ from some books of Canadian Mathematics Competition Problems produced by the University of Waterloo

[©] Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Nfld. A1C 5S7 Phone: (709) 737-8784 • Fax: (709) 737-3010 • Email: mathstat@math.mun.ca • URL: http://www.math.mun.ca