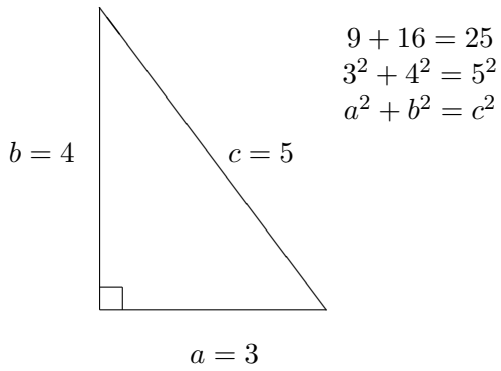
Fermat's Last Theorem Solved!

In Cambridge, England, on the morning of June 23, 1993, an event took place which can be described as the mathematical equivalent of splitting the atom. At the end of a series of three lectures, Professor Andrew Wiles of Princeton University calmly announced in front of an audience of over a hundred experts that he had solved the Fermat conjecture.

Most school children are familiar with the Theorem of Pythagoras which says that if c is the length of the hypotenuse of a right angled triangle, and if a and b are the lengths of the other two sides, then $a^2 + b^2 = c^2$.



Since there are infinitely many right angled triangles with sides of whole number length, there are infinitely many triples of whole numbers satisfying $a^2 + b^2 = c^2$; for example, $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$. In 1637, Pierre de Fermat, a French lawyer and amateur mathematician, was reading a translation of a Greek work in which all the whole number of solutions of the equation $a^2 + b^2 = c^2$ were described. In the margin of this book, Fermat wrote some words which were to baffle the world for 365 years. He wrote, "On the contrary, it is impossible to separate a cube into two cubes, a fourth power into two fourth powers or generally any power above the second into two powers of the same degree. I have discovered a truly marvellous demonstration which this margin is too narrow to contain." In modern notation, Fermat was stating that the equation $a^n + b^n = c^n$ has no non-zero whole number solutions when n is bigger than 2; that is, there are no (non-zero) whole numbers a, b, c with $a^3 + b^3 = c^3$, nor with $a^4 + b^4 = c^4$, nor with $a^5 + b^5 = c^5$, and so on.

Since Fermat's day, the world's finest minds have tried to reconstruct the proof that Fermat claimed to possess, but to no avail prior to Wiles astonishing announcement.

It is difficult for non-mathematicians to understand what mathematical research is, but stop to think about the role that mathematics plays in daily life. Each day, we count and manipulate the decimal system without considering its origins and meaning. Yet there was a time when the decimal system was unknown to humanity. In fact, our number system has its beginnings in the discovery of zero, a profound concept whose discovery represents a major advance in human civilization. All mathematical discoveries are the unearthing of new concepts and their relationships to other concepts and ideas, very much like the discovery of zero.

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The significance of the solution to Fermat's Theorem after such a long time lies not so much in the result itself as in the ideas which have arisen and the theories which have been created throughout the years by the men and women who have attempted to solve the problem. Complex numbers (which are indispensable in science, engineering and space technology) were invented by Carl Friedrich Gauss as part of his proof that $a^3 + b^3 = c^3$ has no non-zero whole number solutions. With time, other such number systems (called algebraic number fields) which pervade all mathematics, were created in efforts to solve other special cases of Fermat's Theorem. Ring theory, algebraic geometry, the theory of elliptic curves and representation theory all had their origins in this remarkable theorem.

Fermat's Last Theorem deserves a special place in the history of civilization. By its simplicity, it tantalized amateurs and professionals alike for nearly four centuries and its proof crowns an edifice composed of the greatest insights of modern mathematics.

What follows is taken from an article by Eric Zorn which appeared in the Chicago Tribune, Tuesday June 29, 1993.

> *News Item (June 23): Mathematicians worldwide were excited and pleased today by the announcement that Princeton University professor Andrew Wiles had finally proved Fermat's Last Theorem, a 365-year old problem said to be the most famous in the field.*

Admittedly, there was rioting and vandalism during the celebration. A few bookstores had windows smashed and shelves stripped, and vacant lots glowed with burning piles of old dissertations. "Math hooligans are the worst," said a Chicago Police Department spokesman, "but we were ready for them this time." When word hit Wednesday that Fermat's Last Theorem had fallen, a massive show of force from law enforcement agencies at universities all around the country headed off a repeat of the festive looting sprees that have become the traditional accompaniment to triumphant breakthroughs in higher mathematics.

Mounted police throughout Hyde Park kept crowds of delirious wizards at the University of Chicago from tipping over cars on the midway as they did in 1976 when Wolfgang Haken and Kenneth Appel cracked the long-vexing "Four Colour Problem." Incidents of textbook-throwing and citizens being pulled from their cars and humiliated with difficult word problems last week were described by the university's math department chairman Bob Zimmer as "isolated". "Most of the celebrations were orderly and peaceful," said Zimmer, "but there will always be a few—usually graduate students—who use any excuse to cause trouble and steal. These are not true fans of Andrew Wiles."

Already there is talk of a dynasty, specifically that next year, Wiles will crack the great unproven Riemann Hypothesis, and after that the Twin Primes Conjecture and the Goldbach Conjecture. "Rie-peat! Rie-peat!", the crowds cry.