THE SIXTEENTH W.J. BLUNDON MATHEMATICS CONTEST*

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1. Solve each of the following equations:

   (a) $\log_{\frac{1}{8}}\left(\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right)\right) = \dfrac{1}{3}$ (b) $\log_3 x + \log_9 x + \log_{27} x = 11$

2. Give three values of x that give the same value of y for

   $$y = 9x^{100} - 4x^{98} + 198.$$

3. If the arithmetic mean of $x^{\frac{1}{2}}$ and $x^{\frac{1}{4}}$ is 6, find x.

4. If $\overline{\text{AB}}$ is a diameter of a circle, a tangent to the circle at D meets AB extended at C, $BC = 5$ and $DC = 8$, find the diameter of the circle.

5. Functions f and g are defined by $f(x) = \sqrt{10 - x}$ and $g(x) = \sqrt{x - 4}$. Find the domain of the composite function $f \circ g$.

6. Solve: $16^x + 32 = 9 \cdot 2^{2x+1}$.

7. How many two-digit numbers are such that the difference of the number and the number with the digits reversed is a perfect square?

8. How many positive real numbers are there that satisfy the equation

   $$x + \log_x 2 = 1\,?$$

9. Find all ordered pairs (x, y) of real numbers that satisfy the system

   $$\sqrt{x - y} = x + y - 7$$
   $$\sqrt{x + y} = x - y - 1\,.$$

10. Semicircle ACB has AB as its diameter. Its centre is O. The tangent at B meets the tangent at C at the point B′. Prove that AC$\|$OB′.

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