THE THIRTY-THIRD W.J. BLUNDON MATHEMATICS CONTEST*

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1. Solve the system

$$\begin{aligned} 3^{2x-y} &= 27 \\ 2^{3x+2y} &= 32 \end{aligned}$$

2. Prove the identity

$$\frac{2016^{-x}}{2016^{-x}+1} + \frac{2016^{x}}{2016^{x}+1} = 1.$$

3. (a) If $\log_3(\log_4(a^3)) = 1$, find a.

   (b) Let $a > 1$. Find all possible solutions for x such that the following equation holds:

$$\log_a x + \log_a(x - 2a) = 2$$

4. (a) Prove that

$$S = 1 + 2 + 3 + \cdots (n-1) + n = \frac{n(n+1)}{2},$$

   for any natural number $n \geq 1$. (Hint: Write the sum backwards and forwards and add the results!)

   (b) Determine the value of n for which

$$2^1 \cdot 2^2 \cdot 2^3 \cdots 2^n = 2^{210}.$$

5. Determine the number of integer values of x such that $\sqrt{2-(1+x)^2}$ is an integer. Fully justify that you have identified the correct number.

6. Find all values of k so that $x^2 + y^2 = k^2$ will intersect the circle with equation

$$(x-5)^2 + (y+12)^2 = 49$$

   at exactly one point.

7. When a two digit number and a three digit number are multiplied, the result is 7777. Find the largest such three–digit number possible.

8. (a) Prove the identity

$$(\sin x)(1 + 2\cos 2x) = \sin(3x)$$

   You may use the identities

$$\sin(x+y) = \sin x \cos y + \sin y \cos x \qquad \cos(x+y) = \cos x \cos y - \sin x \sin y$$

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(b) Suppose n is a positive integer. Prove the identity

$$(1 + 2\cos(2x) + 2\cos(4x) + \ldots + 2\cos(2nx))(\sin x) = \sin((2n+1)x)$$

9. Calculate the value of the product

$$P = \left(1+\frac{1}{2}\right)\cdot\left(1-\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\left(1-\frac{1}{3}\right)\cdot\ldots\cdot\left(1+\frac{1}{n}\right)\left(1-\frac{1}{n}\right)$$

where $n \geq 1$ is a positive integer.

10. Define the function $f(x)$ to be the the largest integer less than or equal to x for any real x. For example $f(1) = 1$, $f(3/2) = 1$, $f(7/2) = 3$, and $f(7/3) = 2$. Let

$$g(x) = f(x) + f(x/2) + f(x/3) + \ldots + f(x/(x-1)) + f(x/x)$$

(a) Calculate $g(4) - g(3)$ and $g(7) - g(6)$.

(b) What is $g(116) - g(115)$?

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