# THE THIRTY-SECOND W.J. BLUNDON MATHEMATICS CONTEST*

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1. Let $f(x) = x^2 + 3x - 40$.

   (a) Solve $f(x) = 0$.

   (b) Suppose a and b are distinct numbers such that $f(a) = f(b)$. Find $a + b$.

   (c) Suppose $f(a) - f(b) = 4$. If a, b are non-negative integers, find all the possible value of a, b.

2. Find the diametrically opposite point on the circle $x^2 + y^2 - 10x + 8y + 16 = 0$ to the point $P = (1, -1)$.

3. Consider the following diagram. If a, b and c denote the radii of circle A, circle B and circle C respectively, find an expression for c in terms of b and b.
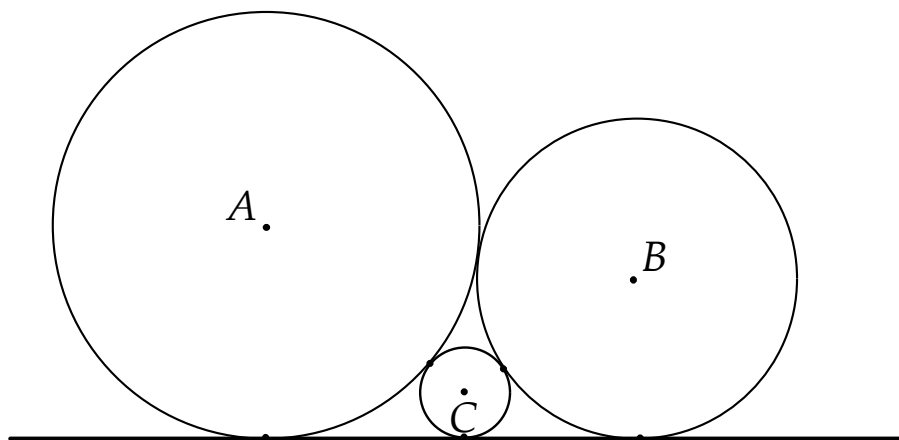
4. Sketch the graph of $|y - x| + |y + x| = 2$.

5. Determine the real values of p and r which satisfy

$$p + pr + pr^2 = 26$$
$$p^2r + p^2r^2 + p^2r^3 = 156$$

6. In the Original Six era of the NHL, one particular season, each team played 20 games (each team played the other 5 teams 4 times each). Each game ended as a win, a loss or a tie (there were no 'overtime losses'). At the end of this certain season, the standings were as below. What were all the possible outcomes for Montreal's number of wins X, losses Y and ties Z ?

Team	Wins	Losses	Ties
Toronto	2	12	6
Boston	6	10	4
Detroit	7	12	1
New York	7	9	4
Chicago	11	7	2
Montreal	X	Y	Z

7. (a) Expand and simplify

$$\left(3^{n/3} - 3^{\frac{n-3}{3}}\right)^3$$

   (b) Use the result of part (a) to calculate the value of

$$\left(3^{4/3} - 3^{1/3}\right)^3 + \left(3^{5/3} - 3^{2/3}\right)^3 + \left(3^{6/3} - 3^{3/3}\right)^3 + \ldots + \left(3^{2006/3} - 3^{2003/3}\right)^3$$

8. The sum of the first n natural numbers, $S = 1 + 2 + \cdots + n$ can be expressed by the formula

$$S = \frac{n(n+1)}{2}.$$

   (a) Suppose the sum of 25 consecutive integers is 500. Determine the smallest of the 25 integers.

   (b) The sum of a set of consecutive integers is 1000. Let m be the first term of this set. Find the smallest positive value of m

9. Prove that there are no real values of x such that

$$2 \sin x = x^2 - 4x + 6$$

10. Two bags, Bag A and Bag B, each contain 9 balls. The 9 balls in each bag are numbered from 1 to 9. Suppose one ball is removed randomly from Bag A and another ball from Ball B. If X is the sum of the numbers on the balls left in Bag A and Y is the sum of the numbers of the balls remaining in Bag B, what is the probability that X and Y differ by a multiple of 4?