THE TWENTY-NINTH W.J. BLUNDON MATHEMATICS CONTEST*

Sponsored by
The Canadian Mathematical Society
in cooperation with
The Department of Mathematics and Statistics
Memorial University of Newfoundland

February 23, 2012

1. (a) Suppose 6 pairs of identical blue socks and 6 pairs of identical black socks are all scrambled in a drawer. How many socks must be drawn out all at once and in the dark to be certain of getting a pair?

   (b) Suppose 3 pairs of identical blue socks, 7 pairs of identical green socks, and 4 pairs of identical black socks are all scrambled in a drawer. How many socks must be drawn out all at once and in the dark to be certain of getting a pair?

2. Find a polynomial equation with integer coefficients whose roots include $\sqrt{2}+\sqrt{3}$.

3. If $xy > 0$, show that $x|y| - y|x| = 0$.

4. If $\log_a(b) + \log_b(a) = 2$, show that $a = b$.

5. Which of $\dfrac{10^{2010}+1}{10^{2011}+1}$ or $\dfrac{10^{2011}+1}{10^{2012}+1}$ is larger? Prove your answer.

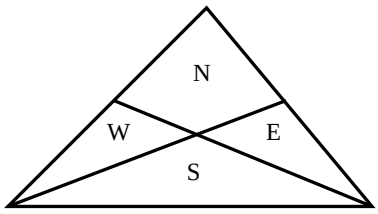
6. If $n!$ is defined as the product of positive integers from 1 to n (so, for example, $5! = 1\cdot2\cdot3\cdot4\cdot5 = 120$), find the final digit in the sum

$$1! + 2! + 3! + \cdots + 2012!.$$

7. Solve $2^{2x+4} + 3^{3x+2} = 4^{x+3}$.

8. A point P is inside a rectangle with vertices labelled A, B, C, and D, consecutively clockwise. Prove that $(PA)^2 + (PC)^2 = (PB)^2 + (PD)^2$.

9. A farmer owns a triangular field, as shown. He reckons 5 sheep can graze in the west field, 10 sheep can graze in the south field, and 8 can graze in the east field. (All sheep eat the same amount of grass.) How many sheep can graze in the north field?



10. A bag is filled with red and blue balls. Before drawing a ball, there is a 1/4 chance of drawing a blue ball. After drawing out a ball, there is now a 1/5 chance of drawing a blue ball. How many red balls are in the bag? (Balls are removed once drawn.)

* *A grant in support of this activity was received from the Canadian Mathematical Society.*
La Société mathématique du Canada a donné un appui financier à cette activité.