THE TWENTY-EIGHTH W.J. BLUNDON MATHEMATICS CONTEST*

Sponsored by
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in cooperation with
The Department of Mathematics and Statistics
Memorial University of Newfoundland

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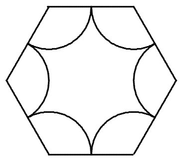
1. Find all the sets of three consecutive integers such that the sum of their squares is 194.

2. Each of the vertices of a rectangle is located on the circle $x^2 + y^2 = 25$. If the perimeter of the rectangle is 28, what is its area?

3. Solve: $x^{\log_{10} x} = \dfrac{x^3}{100}$.

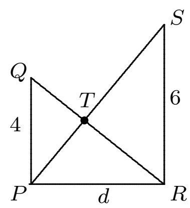
4. A triangle has sides of length 2, 2, and $\sqrt{6} - \sqrt{2}$. Show that the angles are exactly 75 degrees.

5. At each vertex of a regular hexagon, a sector of a circle of radius one-half of the side of the hexagon is removed. Find the fraction of the hexagon remaining.



6. The graphs of $y = \sqrt{3}x^2$ and $x^2 + y^2 = 4$ intersect at A and B.
   (a) Determine the co-ordinates of A and B.
   (b) Determine the length of the minor arc AB of the circle.

7. A pair of telephone poles d metres apart is supported by two cables which run from the top of each pole to the bottom of the other. The poles are 4 m and 6 m tall. Determine the height above the ground of the point T, where the two cables intersect. What happens to this height as d increases?



8. The equation $x^3 - px^2 + qx - r = 0$ has roots which are consecutive integers. Determine r and q in terms of p.

9. Show that if $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ and $\dfrac{a}{x} + \dfrac{b}{y} + \dfrac{c}{z} = 0$, then $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

10. Show that if $abc \neq 0$ and $(x^2 + y^2 + z^2)(a^2 + b^2 + c^2) = (ax + by + cz)^2$, then $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$.

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