# THE TWENTY-SEVENTH W.J. BLUNDON MATHEMATICS CONTEST*

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1. Given that $2x^2 + 5x - 3$ is a factor of $P(x) = 8x^4 + 8x^3 - 40x^2 + ax + b$, find the product of the zeros of $P(x)$.

2. An equilateral triangle is inscribed inside a circle of radius $r$. Find the area of the triangle.

3. Find all pairs $(x, y)$ of positive integers such that

$$2xy - 4x^2 + 12x - 5y = 5\,.$$

4. If $O(0, 0)$, $A(2, 4)$ and $B(6, 2)$ are vertices of a triangle, find the point $P$ on $AB$ so that $OP$ divides the triangle into two triangles of equal area.

5. Find the area of the triangle with vertices $O(0, 0)$, $A(a, b)$ and $B(c, d)$. You may assume that $a$, $b$, $c$ and $d$ are positive.

6. Find two points on the parabola $y = x^2$ such that the distance between them is 5 and the slope of the line joining them is $\frac{4}{3}$.

7. Find all pairs $(x, y)$ of integers that satisfy the equation

$$y^2 = x^2 + 2x + 6\,.$$

8. Find the minimum value of $y = (x - a)^2 + (x - b)^2$, where $a$ and $b$ are constants.

9. A line with positive slope passes through the origin and is tangent to the circle with center $(0, 6)$ and radius 2. Find the equation of the line and the point of intersection.

10. Find all positive integers $a$ and $b$ that satisfy the equation

$$\frac{1}{a} + \frac{a}{b} + \frac{1}{ab} = 1\,.$$

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