THE TWENTY-SIXTH W.J. BLUNDON MATHEMATICS CONTEST[*]

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1. Prove that if $ac + d = ad + c$ and $a \neq 1$, then $c = d$.

2. If $xy = 2$ and $\dfrac{1}{x^2} + \dfrac{1}{y^2} = 3$, find all possible values of $x + y$.

3. If John gets 97 on his next math test, his average will be 90. If he gets 73, his average will be 87. How many math tests has John already taken?

4. A piggy bank contains 100 coins consisting of nickels, dimes and quarters. If the total amount in the piggy bank is \$9.50, find the maximum number of quarters that the piggy bank can contain.

5. A palindrome is a word or number that reads the same backwards and forwards. For example, 1991 is a palindromic number. How many palindromic numbers are there between 1 and 999,999 inclusive?

6. At a party each man shook hands with everyone except his spouse, and no handshakes took place between women. If 13 married couples attended, how many handshakes were there among these 26 people?

7. Show that there are no real values of x and y such that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{x+y}$.

8. A number like 12321 that reads the same backwards and forwards is called a palindrome. Prove that all four digit palindromes are divisible by 11.

9. A line with positive slope passes through the origin and intersects the parabola $y = x^2 + 4$ at exactly one point. Find the equation of the line.

10. Solve $\sqrt[3]{x+9} - \sqrt[3]{x-9} = 3$.