THE TWENTY-FOURTH W.J. BLUNDON MATHEMATICS CONTEST*

Sponsored by
The Canadian Mathematical Society
in cooperation with
The Department of Mathematics and Statistics
Memorial University of Newfoundland

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1. Determine how many integers from 1 to 10,000 inclusive are divisible by 4 but not by 5.

2. (a) Factor completely: $a^5 + 4b^5 - a^2b^3 - 4a^3b^2$.
   (b) Given that $a = \dfrac{x}{x^2+y^2}$ and $b = \dfrac{y}{x^2+y^2}$, solve for x and y in terms of a and b.

3. For $ABCD$ a square, let M and N be the midpoints of BC and CD, respectively, and let θ be angle MAN. Find $\sin\theta$.

4. Find the equation of the tangent line to the curve $2x^2 + 2y^2 - 6x + 2y - 5 = 0$ at the point $(\frac{7}{2}, \frac{1}{2})$.

5. Find all solutions to the system of equations $x + y = 2$ and $\dfrac{x^2}{y} + \dfrac{y^2}{x} = \dfrac{14}{3}$.

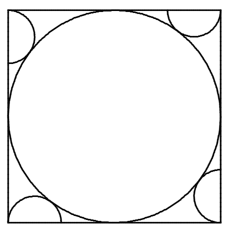
6. Ten points are scattered inside a 9×12 rectangle. Prove that at least two of the points are within 5 units of each other.

7. Find three complex numbers z such that $z^3 = i$.

8. A polynomial $P(x)$ has $P(1) = 3$ and $P(2) = 5$. Find the remainder when $P(x)$ is divided by $(x-1)(x-2)$.

9. Consider the sequence $a_1, a_2, a_3, \ldots$ of positive numbers in which each term is one more than its predecessor, divided by the term before that. For example,
   $$2,\ 5,\ 3,\ \frac{4}{5},\ \frac{3}{5},\ 2,\ 5,\ 3,\ \ldots$$
   Prove that any such sequence always has period 5.

10. A circle is inscribed in a square. Four semicircles with their flat sides along the edge of the square and tangent to the circle are inscribed in each of the four spaces between the square and circle. What is the ratio of the area of the circle to the total area of the four semicircles?



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