THE TWENTY-THIRD W.J. BLUNDON MATHEMATICS CONTEST*

February 22, 2006

1. If $\log_a x = \log_b y$, show that each is also equal to $\log_{ab} xy$.

2. In how many ways can 20 dollars be changed into dimes and quarters, with at least one of each coin used?

3. If one of the women at a party leaves, then 20% of the people remaining at the party are women. If, instead, another woman arrives at the party, then 25% of the people at the party are women. How many men are at the party?

4. Find two factors of $2^{48} - 1$ between 60 and 70.

5. The yearly changes in the population census of a town for four consecutive years are, respectively, 25% increase, 25% increase, 25% decrease, and 25% decrease. Find the net percent change to the nearest percent over the four years.

6. If $x + y = 5$ and $xy = 1$, find $x^3 + y^3$.

7. The point $(4, 1)$ is on the line that passes through the point $(4, 1)$ and is perpendicular to the line $y = 2x + 1$. Find the area of the triangle formed by the line $y = 2x + 1$, the given perpendicular line, and the x-axis.

8. An arbitrary point is selected inside an equilateral triangle. From this point perpendiculars are dropped to each side of the triangle. Show that the sum of the lengths of these perpendiculars is equal to the length of the altitude of the triangle.

9. Find all positive integer triples (x, y, z) satisfying the equations

$$x^2 + y - z = 100 \quad \text{and} \quad x + y^2 - z = 124.$$

10. How many roots are there to the equation $\sin x = \dfrac{x}{100}$?