THE NINETEENTH W.J. BLUNDON MATHEMATICS CONTEST[*]

February 20, 2002

1. Five years ago Janet was one sixth of her mother's age. In thirteen years she will be half her mother's age. What is Janet's present age?

2. If $a + b + c = 0$, prove that $a^3 + b^3 + c^3 = 3abc$.

3. A certain rectangle has area 6 and diagonal of length $2\sqrt{5}$. What is its perimeter?

4. Find all positive numbers x such that $x^{x\sqrt{x}} = (x\sqrt{x})^x$.

5. Rationalize the denominator: $\dfrac{1}{\sqrt{2} + \sqrt{3} + \sqrt{6}}$.

6. Points A and B are on the parabola $y = 2x^2 + 4x - 2$. The origin is the midpoint of the line segment joining A and B. Find the length of this line segment.

7. If $\log_{125} 2 = a$ and $\log_9 25 = b$, find $\log_8 9$ in terms of a and b.

8. Point P lies in the first quadrant on the line $y = 2x$. Point Q is a point on the line $y = 3x$ such that PQ has length 5 and is perpendicular to the line $y = 2x$. Find the point P.

9. For what conditions on a and b is the line $x + y = a$ tangent to the circle $x^2 + y^2 = b$?

10. In $\triangle ABC$, we have $\angle ACB = 120$ degrees, $AC = 6$ and $BC = 2$. The internal bisector of $\angle ACB$ meets the side AB at the point D. Determine the length of the line segment CD.