

# THE EIGHTEENTH W.J. BLUNDON MATHEMATICS CONTEST\*

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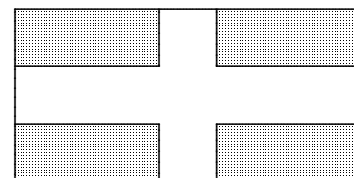
in cooperation with  
The Department of Mathematics and Statistics  
Memorial University of Newfoundland

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- (a) At a meeting of 100 people, every person shakes hands with every other person exactly once. How many handshakes are there in total?  
(b) How many four digit numbers are divisible by 5?
- Show the  $n^2 + 2$  is divisible by 4 for no integer  $n$ .
- Prove that the difference of squares of two odd integers is always divisible by 8.
- The inscribed circle of a right triangle  $ABC$  is tangent to the hypotenuse  $AB$  at  $D$ . If  $AD = x$  and  $DB = y$ , find the area of the triangle in terms of  $x$  and  $y$ .
- Find all integers  $x$  and  $y$  such that

$$2^x + 3^y = 3^{y+2} - 2^{x+1}.$$

- Find the number of points  $(x, y)$ , with  $x$  and  $y$  integers, that satisfy the inequality  $|x| + |y| < 100$ .
- A flag consists of a white cross on a red field. The white stripes are of the same width, both vertical and horizontal. The flag measures  $48\text{cm} \times 24\text{cm}$ . If the area of the white cross equals the area of the red field, what is the width of the cross?



- Solve:  $\frac{x+1}{2+\sqrt{x}} - \frac{1}{2-\sqrt{x}} = 3$ .

- Let  $P(x)$  and  $Q(x)$  be polynomials with “reversed” coefficients

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0,$$
$$Q(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-2} x^2 + a_{n-1} x + a_n$$

where  $a_n \neq 0$ ,  $a_0 \neq 0$ . Show that the roots of  $Q(x)$  are the reciprocals of the roots of  $P(x)$ .

- If  $1997^{1998}$  is multiplied out, what is the units digit of the final product?



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