THE SEVENTEENTH W.J. BLUNDON MATHEMATICS CONTEST*

Sponsored by
The Canadian Mathematical Society

in cooperation with
The Department of Mathematics and Statistics
Memorial University of Newfoundland

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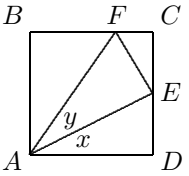
1. The numbers represented by $1, 1+2, 1+2+3, 1+2+3+4$, etc., are called **triangular numbers**. The nth triangular number t_n $(n \geq 1)$ can be found by the formula $t_n = \dfrac{n(n+1)}{2}$. For example, $t_3 = \dfrac{3(4)}{2} = 6$.
   (a) Evaluate $\sqrt{t_{10} + t_{11}}$.
   (b) Show that $t_n + t_{n+1}$ is always a perfect square.

2. For any two positive integers m and n (where $m > n$), the values of $m^2 - n^2$, $2mn$ and $m^2 + n^2$ will represent the lengths of the sides of a right triangle.
   (a) Determine m and n that will produce a right triangle with sides of lengths 3, 4 and 5.
   (b) Find the area of a right triangle with a hypotenuse of 34 units and legs of integral lengths.

3. A post office sells only 46 cents and 55 cents stamps. Suppose that you need some stamps and you want to purchase an exact number of dollars worth of stamps. What is the minimum number of dollars that you could spend?

4. Find the conditions for the positive integers A, B and C to satisfy $\dfrac{A^3 + B^3}{A^3 + C^3} = \dfrac{A + B}{A + C}$.

5. $ABCD$ is a square of sides length 4, E is the midpoint of CD, and $AE \perp EF$, as shown. If x and y are the measures of $\angle EAD$ and $\angle FAE$ respectively, prove that $x = y$.



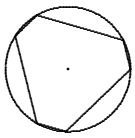
6. Two parallel lines cross both the x and y axes. The intercepts are 3 cm apart on the x–axis and 4 cm apart on the y–axis. How far apart are the lines?

7. If $x = 3 + \dfrac{1}{3 + \frac{1}{x}}$ and $y = 3 + \dfrac{1}{3 + \frac{1}{3 + \frac{1}{y}}}$, find the value(s) of $|x - y|$.

8. Consider the quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$ and a, b and c are integers. Prove that the discriminant can never equal 99.

9. Tom and Don both made the same trip of 220 km, Don making the trip in 20 minutes less time, averaging 5 km/hr faster than Tom. What was the average speed of both drivers?

10. A hexagon, H, is inscribed in a circle, and consists of three segments of length 1 and three segments of length 3. Find the area of H.



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