THE THIRTY-EIGTH W.J. BLUNDON MATHEMATICS CONTEST*

Sponsored by
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The Department of Mathematics and Statistics
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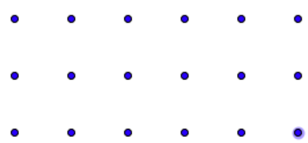
1. Let $A = \sqrt{19} + \sqrt{99}$ and $B = \sqrt{20} + \sqrt{98}$. Determine which number is larger and justify your conclusion.

2. Trees had been planted in a park to form a square lattice with 100 rows of 100 trees in each row. A portion of the lattice is shown in the figure.



Later, several trees were cut in such a way that one cannot see any other stumps while standing on any stump. What could be the maximum number of stumps?

3. The following equation has two real roots:

$$x^2 + 18x + 30 - 2\sqrt{x^2 + 18x + 45} = 0.$$

Find the product of these roots.

4. Find all x with $1/2 \le x \le 1$ such that $4^{\cos(2x)} + 4^{\cos^2 x} = 3$.

Hint: You may use the formula $\cos(2x) = 2\cos^2 x - 1$.

5. An isosceles trapezoid $ABCD$ ($AB = CD$ and $AD||BC$) can be cut along its diagonal to form 2 isosceles triangles. Find the measure of all angles of the trapezoid. Describe all the possible cases.

6. Suppose that we have a deck of cards of various colours. We draw a card, note its colour, put it back in the deck, reshuffle the deck and then draw another card.

   (a) Let R, B, G, Y be the probabilities to draw respectively a red, blue, green, or yellow card. What is the probability P of drawing four cards in a row, one of each of these colours?

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(b) Calculate P given that

$$R = \frac{(\sqrt{5}+\sqrt{6}+\sqrt{7})}{10}, \quad B = \frac{(\sqrt{5}+\sqrt{6}-\sqrt{7})}{30},$$
$$G = \frac{(\sqrt{5}-\sqrt{6}+\sqrt{7})}{30}, \quad Y = \frac{(-\sqrt{5}+\sqrt{6}+\sqrt{7})}{30}.$$

Give your answer in the form of an irreducible fraction.

7. Let $x = 8^9 + 7^9 + 6^9$. Suppose that x is divided by 5. What is the remainder?

8. In an unnamed country, Donald and Joe are running for president. There are 3 states. Each state consists of 3 counties. Each county has 3 cities, and each city has 3 wards. Each ward has 3 electors who cast votes. To win a ward, a candidate must win 2/3 of the electors; to win a city, one must win 2/3 of the wards; to win a county, one must win 2/3 cities; and to win a state, you have to win 2/3 of the counties, and finally to win the election, you must win 2/3 of the states. Abstaining from voting is not allowed.

   (a) What is the smallest number of elector votes Donald must receive to win the election? What percentage of the total popular vote is this?

   (b) What is the smallest number of total votes Joe needs to guarantee a victory?

9. Given three distinct numbers (labelled by a, b, c), one generates a new number by the following rules:

   - each of the numbers must be used once and only once;
   - each of the operations of addition $+$, multiplication $\times$, and brackets () may be used any number of times, or not at all.

   For example, $(a+b)\times c$ and $a+b+c$ are allowed, but $a+a\times b+c$ and $a+c$ are not.

   (a) What is the **maximum** number of different numbers that can be generated from (a, b, c) according to these rules?

   (b) Repeat the above problem but now for **four** distinct numbers (a, b, c, d).

10. Suppose that x and y are **integers** that satisfy

$$y^2 + 3x^2y^2 = 30x^2 + 517.$$

Determine $a = 3x^2y^2$.

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