THE THIRTY-SEVENTH W.J. BLUNDON MATHEMATICS CONTEST*

Sponsored by
The Canadian Mathematical Society
in cooperation with
The Department of Mathematics and Statistics
Memorial University of Newfoundland

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1. How many real solutions does the following equation have?
$$\frac{1}{x} = \sqrt{x^2 - 2}.$$

2. A rectangle has 4 positive numbers placed at its vertices. Each number is greater than or equal to the average of the two numbers at the adjacent vertices. Prove that all tour numbers are in fact equal.

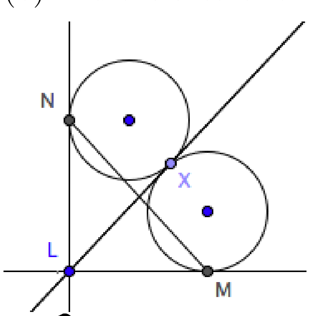
3. After another global financial crisis, Newfoundland introduces a new currency consisting of only 3 and 5 dollar bills. To save resources, no coins or other bills are created, and stores cannot give any change. What prices are not allowed?

4. (a) Find positive integers a and b such that $a^2 - b^2 = 2^3 = 8$.
   (b) Find positive integers $a = a(N)$ and $b = b(N)$ (that is, find a, b as functions of N) such that $a^2 - b^2 = N^3$ for any $N \geq 1$.

5. Bob arranged N marbles in 2 squares of sizes $a \times a$ and $c \times c$ respectively. Alice rearranged the same number of marbles in a square of size $b \times b$ and a rectagle 3×19. Given that a, b, c are consecutive odd numbers, find N.

6. Maggie takes each number from 1 to 1000 and replaces it with the sum of its digits. For example, $123 \to 1+2+3 = 6$ or $95 \to 9+5 = 14$. Then she does the same with each resulting number up until she gets 1000 single digit numbers. Let m be the number of 1's and n be the number of 2's among the resulting single digit numbers. Find $m - n$.

7. Two circles with the same radius are tangent to each other at a point X. Tangents from a point L are drawn to the two circles, hitting them at X and at M and N, as shown in the diagram. Given that $\angle MLN$ is a right angle and $LX = 2$cm,
   (a) determine the length of MN;
   (b) find the area of the quadrilateral $LNXM$.



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8. Let $ABCDEF$ be a convex hexagon such that all its inner angles are the same, but the sides are of distinct lengths.

   a) Find the measure of the hexagon's angles.

   b) Prove that opposite sides of the hexadon are pairwise parallel:

   $AB||DE, BC||EF$ and $CD||FA$.

   c) Prove that all three differences of lengths of opposite sides are equal, namely:

$$AB - DE = CD - FA = EF - BC$$

9. Suppose there is a group of 8 students whose names are placed in alphabetical order in a list, called List A. Four students are randomly chosen from this group and again their names are placed in alphabetical order in List B. Find the probability that the 3rd person in the original list of 8 students (List A) is first in the list of 4 students (List B).

10. Vladimir and Donald play n rounds of a game where $n = 1, 2, 3, \ldots$. In each round, there are no ties, and the winner receives 2^{n-1} dollars from the loser. After 30 rounds are played, Vladimir has a profit of 2021 dollars (and Donald has lost 2021 dollars). How many rounds did Vladimir win, and which rounds were they?
    Hint: you might find the following identity, valid for x real, useful:

$$1 + x + x^2 + \ldots + x^{n-1} = \frac{x^n - 1}{x - 1}$$

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