Graduate Seminar

Ms. Guoli Xia
Memorial University

Thursday, January 31, 2019
1 - 2 pm in HH-3017

When is every linear transformation a sum of an idempotent one and a locally nilpotent one?

Abstract:

An endomorphism f of a module M is locally nilpotent if for any $x \in M$, $f^n(x) = 0$ for some $n > 0$. In this note, it is shown that for a vector space V over a division ring D, every linear transformation of V is a sum of an idempotent linear transformation and a locally nilpotent linear transformation if and only if $D \cong \mathbb{F}_2$. This can be seen as an answer to the "local" version of a question raised by Breaz et al. on nil-cleanness of the ring of linear transformations of an infinite dimensional vector space. As an extension, it is proved that for a semisimple module M over a ring R with $R/J(R)$ Boolean, every endomorphism of M is a sum of an idempotent endomorphism and a locally nilpotent endomorphism.