THE TWENTY-FIFTH W.J. BLUNDON MATHEMATICS CONTEST*

Sponsored by
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in cooperation with
The Department of Mathematics and Statistics
Memorial University of Newfoundland

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1. Two sides of an isosceles triangle are 5 cm each and the area of the triangle is 12 cm^2. Find all possible values for the length of the third side.

2. Solve: $8^x + 16 \cdot 8^{-x} = 17$.

3. If $x^3 + y^3 = 10(x + y)$ and $x^2 + y^2 = 30$, find xy.

4. For the points $A(1, 4)$, $B(3, 7)$, $C(7, 8)$ and $D(10, 2)$, find the area of the quadrilateral $ABCD$.

5. A rectangle $ABCD$ has sides $AB = CD = 6$ and $AD = BC = 8$. Two equal circles of radius r are inside this rectangle. One is tangent to AB and to BC, and the other is tangent to CD and to DA. The two circles are externally tangent to each other. Determine the exact value of r.

6. When one kilogram of salt is added to a solution of salt and water, the solution becomes $33\frac{1}{3}\%$ salt by mass. When one kilogram of water is added to this new solution, the resulting solution is 30% salt by mass. Find the percentage of salt in the original solution.

7. How many pairs of integers (x, y) satisfy the equation $x^4 + \dfrac{100}{y^4} = \dfrac{101x^2}{y^2}$?

8. Show that the circles with equations $x^2 + y^2 + 2x - 8y + 8 = 0$ and $x^2 + y^2 + 10x - 2y + 22 = 0$ are tangent.

9. Find all real numbers a such that the polynomials $x^3 + ax^2 + 1$ and $x^3 + x^2 + a$ have at least one zero in common.

10. Prove that the sum of cubes of three consecutive integers is divisible by 9.

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