The Seventeenth W.J. Blundon Contest – Solutions

1. (a) $\sqrt{t_{10}+t_{11}} = \sqrt{\frac{10(11)}{2}+\frac{11(12)}{2}} = \sqrt{55+66} = \sqrt{121} = 11$

(b) $t_n + t_{n+1} = \dfrac{n(n+1)}{2} + \dfrac{(n+1)(n+2)}{2} = \dfrac{(n+1)[n+(n+2)]}{2} = \dfrac{(n+1)(2n+2)}{2}$
$= (n+1)(n+1) = (n+1)^2$

2. (a) Since $(m-n)^2 > 0$, it follows that $m^2 - 2mn + n^2 > 0$, and hence $m^2+n^2 > 2mn$. Also clearly $m^2+n^2 > m^2-n^2$. So the side of of length m^2+n^2 is the hypotenuse. Then for a right triangle with sides of lengths 3, 4 and 5, we must have $m^2+n^2=5$. The only positive integer solutions to this with $m > n$ are $m=2,\ n=1$.

(b) m^2+n^2 is the length of the hypotenuse. So
$$m^2+n^2 = 34 \quad \Rightarrow \quad m=5\,,\ n=3$$
The other sides have lengths of
$$5^2-3^2=16 \quad \text{and} \quad 2(5)(3)=30$$
So
$$\text{Area of triangle } = \frac{1}{2}bh = \frac{1}{2}(16)(30) = 240$$

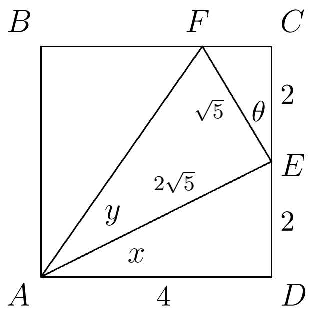
3. Let m be the number of 46¢stamps and n be the number of 55¢stamps. Since $46m+55n$ must be an even dollar value, m must be a multiple of 5.

If $m=0$, then $n=20$ and the total cost would be \$11.
If $m=5$, then $n=14$ and the total cost would be \$10.
If $m=10$, then $n=8$ and the total cost would be \$9.
If $m=15$, then $n=2$ and the total cost would be \$8.

Since the cost of twenty 46¢stamps is \$9.20, the minimum purchase is \$8.

4.
$$\frac{A^3+B^3}{A^3+C^3} = \frac{A+B}{A+C}$$
$$\frac{(A+B)(A^2-AB+B^2)}{(A+C)(A^2-AC+C^2)} = \frac{A+B}{A+C}$$
$$A^2-AB+B^2 = A^2-AC+C^2$$
$$A(C-B) = C^2-B^2$$
$$A(C-B) = (C-B)(C+B)$$
$$C=B \text{ or } A=C+B$$

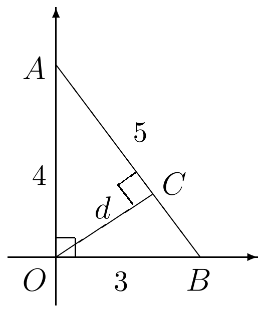
5.



$\theta + 90\deg + (90\deg - x) = 180\deg \quad \Rightarrow \quad \theta = x$

So triangles AED and EFC are similar. So $FC=1$. Then
$$EF=\sqrt{5} \quad \text{and} \quad AE = 2\sqrt{5}$$
So triangle AEF is similar to triangle ADE, and so $x=y$.

6.



Without loss of generality we may assume that one line passes through the origin. Thus we are looking for the altitude OC of the 3–4–5 right triangle AOB. Now triangles OAC and OAB are similar. So

$$\frac{d}{4} = \frac{3}{5} \quad \Rightarrow \quad d = \frac{12}{5}$$

7. $x = 3 + \dfrac{1}{3+\frac{1}{x}} = 3 + \dfrac{x}{3x+1} = \dfrac{10x+3}{3x+1} \quad \Rightarrow \quad \begin{aligned} 3x^2 - 9x - 3 &= 0 \\ x^2 - 3x - 1 &= 0 \\ x &= \frac{3 \pm \sqrt{13}}{2} \end{aligned}$

$$y = 3 + \frac{1}{3+\frac{1}{3+\frac{1}{y}}} = 3 + \frac{1}{3+\frac{y}{3y+1}} = 3 + \frac{3y+1}{10y+3} = \frac{33y+10}{10y+3} \quad \Rightarrow \quad \begin{aligned} 10y^2 - 30y - 10 &= 0 \\ y^2 - 3y - 1 &= 0 \\ y &= \frac{3 \pm \sqrt{13}}{2} \end{aligned}$$

So $|x - y| = \sqrt{13}$ or 0

8. $ax^2 + bx + c = 0$

$$D = b^2 - 4ac$$

If $b^2 - 4ac = 99$, then b must be odd, say $b = 2n + 1$. Then

$$\begin{aligned} (2n+1)^2 - 4ac &= 99 \\ 4n^2 + 4n + 1 - 4ac &= 99 \\ 4(n^2 + n - ac) &= 98 \end{aligned}$$

But 4 is not a factor of 98. So $b^2 - 4ac = 99$ is not possible

9. $r =$ Tom's rate
$r + 5 =$ Don's rate

$\dfrac{220}{r} =$ Tom's time, $\quad \dfrac{220}{r} - \dfrac{1}{3} =$ Don's time

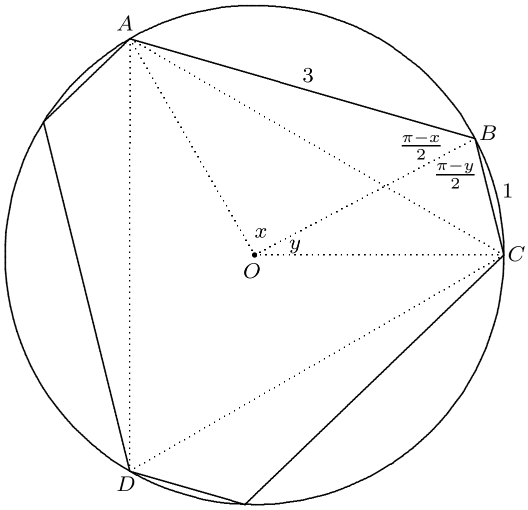
$$\text{Don's distance} = \text{Don's rate} \times \text{Don's time}$$

$$220 = (r+5)\left(\frac{220}{r} - \frac{1}{3}\right)$$

$$\text{Solve:} \quad r = 55$$

So Tom's average speed is 55 km/hr and Don's is 60 km/hr.

10. Consider the figure below.



Clearly $x+y=\dfrac{2\pi}{3}$. Therefore

$$\angle ABC = \frac{\pi-x}{2}+\frac{\pi-y}{2} = \pi - \frac{x+y}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Then use the law of cosines to obtain

$$\begin{aligned} AC^2 &= AB^2 + BC^2 - 2(AB)(BC)\cos\angle ABC \\ &= 3^3 + 1^2 - 2(3)(1)\cos\frac{2\pi}{3} \\ &= 9 + 1 - 6\left(-\tfrac{1}{2}\right) \\ &= 13 \\ AC &= \sqrt{13} \end{aligned}$$

Then the required area is

$$\begin{aligned} \text{Area} &= \text{Area of } \triangle ADC \ + \ 3 \text{ times area of } \triangle ABC \\ &= \frac{1}{2}\sqrt{13}\left(\sqrt{13}\sin\frac{\pi}{3}\right) \ + \ 3\left(\frac{1}{2}(1)\,3\sin\frac{\pi}{3}\right) \\ &= \frac{13}{2}\cdot\frac{\sqrt{3}}{2} + \frac{9}{2}\cdot\frac{\sqrt{3}}{2} = \frac{22}{2}\cdot\frac{\sqrt{3}}{2} = \frac{11\sqrt{3}}{2} \end{aligned}$$