PROC 5071: Process Equipment Design I

Mixing Equipment

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1 Fundamentals of agitation and mixing

1.1 What is mixing? What is agitation? Are they different?

- Mixing refers to the process to randomly distribute two or more initially separate phases into and through one another.
- Agitation refers to induced motion of a material in a specified way, usually in a circulating pattern inside some sort of container.
- Often, the purpose of agitation is to keep a mixture (that has already been mixed) in the proper mixed state.

1.2 Purposes of agitation and mixing

- Blending two miscible liquid
  - ethyl alcohol and water
- Dissolving solids in liquids
  - salt in water
- Dispersing a gas in a liquid as fine bubbles
  - oxygen from air in a suspension of microorganisms for fermentation
  - oxygen in sludge in waste treatment using the activated sludge process
- Suspending fine solid particles in a liquid
  - solid catalytic particles and hydrogen bubbles dispersion in catalytic hydrogenation of a liquid

- Increase heat transfer
  - transfer of heat to fluid in a vessel from a coil or jacket in the vessel wall

1.3 Examples of industrial application of agitation

- Paint industry
  - pigment suspension
  - maintaining suspension
  - storage
- Sugar industry
  - starch converter
  - enzyme conversion
  - dissolving
- Water treatment
  - lime slaking tank
  - converting CaO into $Ca(OH)_2$
  - reaction

- Separation
  - catalytic hydrogenation
  - mass transfer between liquid and solid particles

1.4 Coverage of agitation in different courses

- Fluid dynamics
  - flow pattern of fluids
- Heat transfer
  - effect of liquid motion on heat transfer coefficients
- Mass transfer
  - mass transfer to drops, bubbles and liquid particles
- Unit operations
  - equipment and operations

1.5 Our focus in this course

- Choice and sizing of equipment
- Determining power requirement
- Determining optimum operating conditions

2 Typical design of agitated vessels

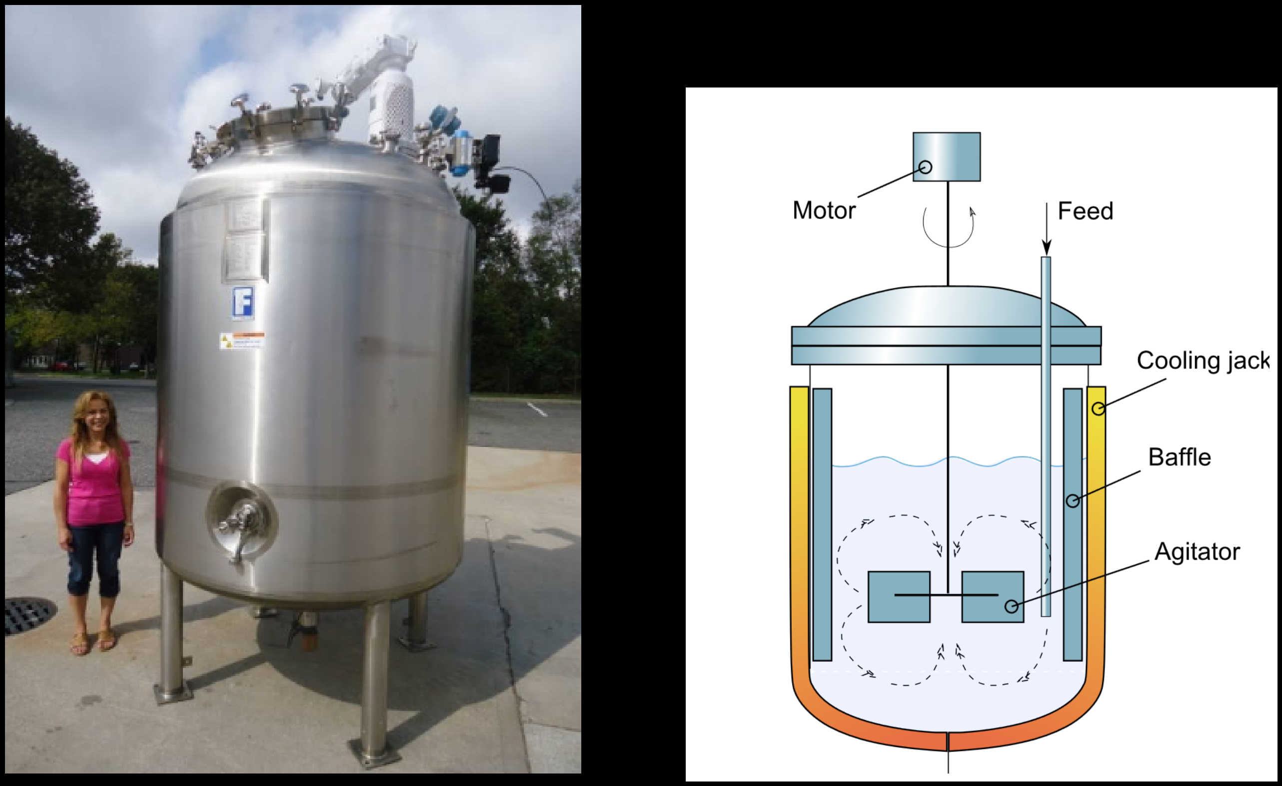

Figure 1: Pictorial and schematic of a typical agitated vessel.

- Cylindrical with a vertical axis

- Usually close at the top; sometimes open to air
- Bottom is rounded, not flat - to eliminate sharp corners or regions into which fluid currents could not penetrate
- Liquid depth is equal to the diameter of the tank
- An impeller is mounted on an overhung shaft driven by a motor
- Baffles are often used to reduce tangential motion
- Accessories - inlet and outlet lines, coils, jackets, wells for sensors

Agitation vessels differ primarily depending on the type of the impeller used.

3 Impellers

3.1 Purpose of an impeller

- The main purpose of the impeller is to cause the liquid to circulate through the vessel and eventually return to the impeller.

3.2 Types of impellers

- Depending on the direction of the induced flow, impellers are of two types
  1. Axial-flow impellers: those that generate current parallel with the axis
  2. Radial-flow impellers: those that generate current in a radial or tangential direction
- Different types of impellers are needed depending on the liquid viscosity
  - For low to moderate viscosity liquids
    1. Propellers
    2. Turbines
    3. High efficiency impellers

- For high viscosity liquids
  1. Helical impellers
  2. Anchor agitators

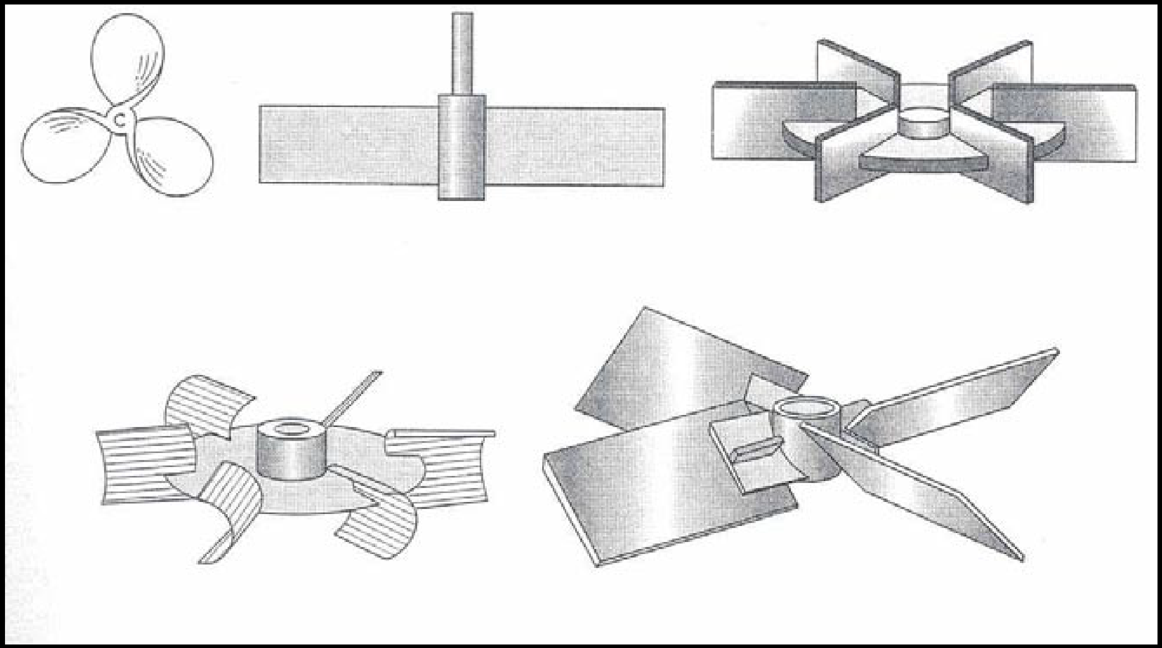

Figure 2: Different types of impellers. From top left clockwise (i) three-blade turbine propeller (ii) simple straight-blade turbine (iii) disk turbine (iv) concave-blade CD-6 impeller (v) pitched-blade turbine

3.3 Propeller

- axial flow
- high speed - small:1150-1750 rpm, large: 400-800 rpm
- direction of rotation - force the liquid downwards
- traces out a helix in the liquid
- one full revolution move the liquid longitudinally a fixed distance
- $pitch = \frac{distance\ liquid\ moves\ with\ 1\ revolution}{propeller\ diameter}$
- Square pitch - pitch $=$1.
- most common - three blade with square pitch
- rarely exceeds $18\ in$ in diameter
- two or more propellers may be mounted on the same shaft

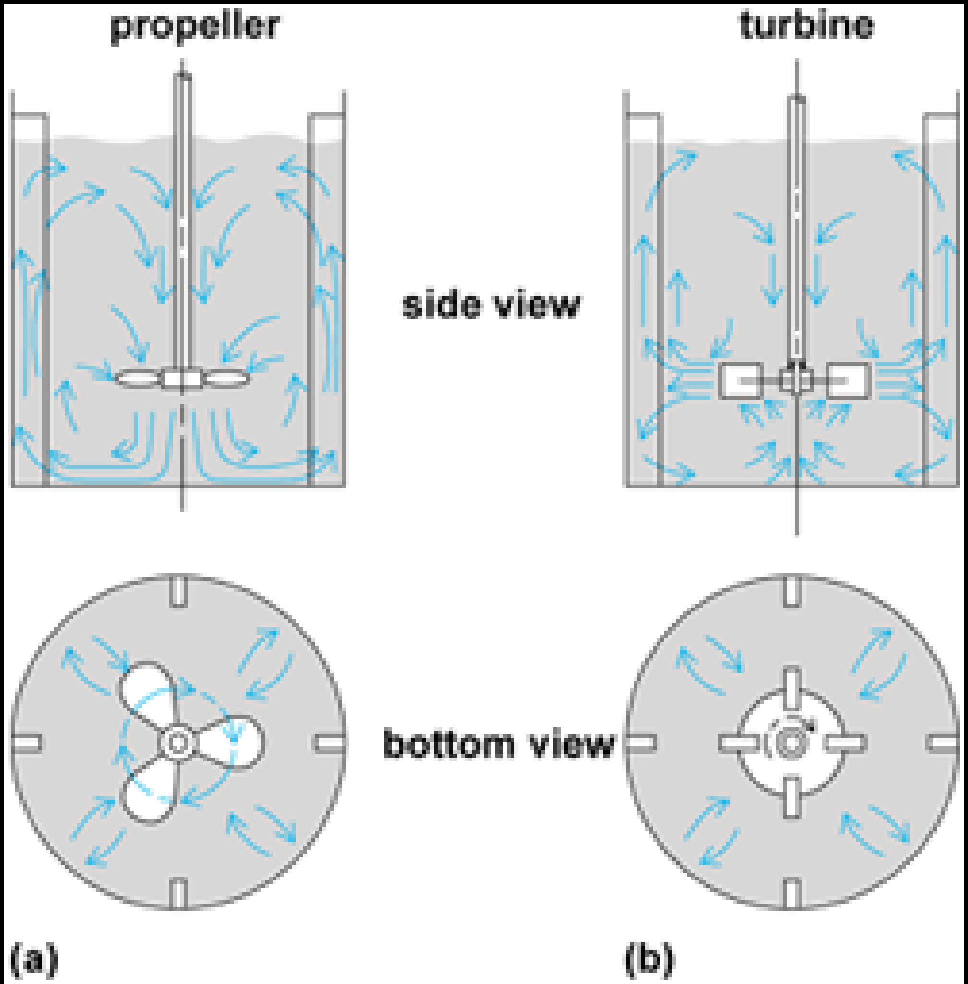

Figure 3: Liquid flow patterns in agitated vessels

3.4 Turbines

- Straight blade
  - pushes liquid radially and tangentially
  - almost no vertical motion at the impeller
  - typical speed is 20-150 r/min
- Disk turbine
  - multiple straight blade mounted on a horizontal disk
  - useful for dispersing gas in a liquid
  - blades may have different shape e.g. concave blade disk turbine
- Pitched blade
  - good when overall circulation is important
  - some axial flow in addition to radial flow

3.5 High efficiency impellers

- variations of pitched blade impellers
- more uniform axial flow and better mixing
- modified shapes of blade

3.6 Hellical ribbon

Figure 4: Impellers for high-viscosity liquids (i) double-flight helical-ribbon (ii) anchor impeller

- Used for highly viscous solution
- Operates at low rpm in the laminar region.
- The liquid moves in a tortuous flow path down the centre and up.
- The diameter of the helix is very close to the diameter of the tank.

3.7 Anchor impellers

- Anchor impeller provides good agitation near the floor and wall of the tank.
- It creates no vertical motion.
- Less effective as in mixing.
- Provide good heat transfer to and from vessel wall.
- Used for viscous liquids where deposits on walls can occur.
- Used to process starch pastes, paints, adhesives and cosmetics.

4 Theoretical development

4.1 Nomenclature

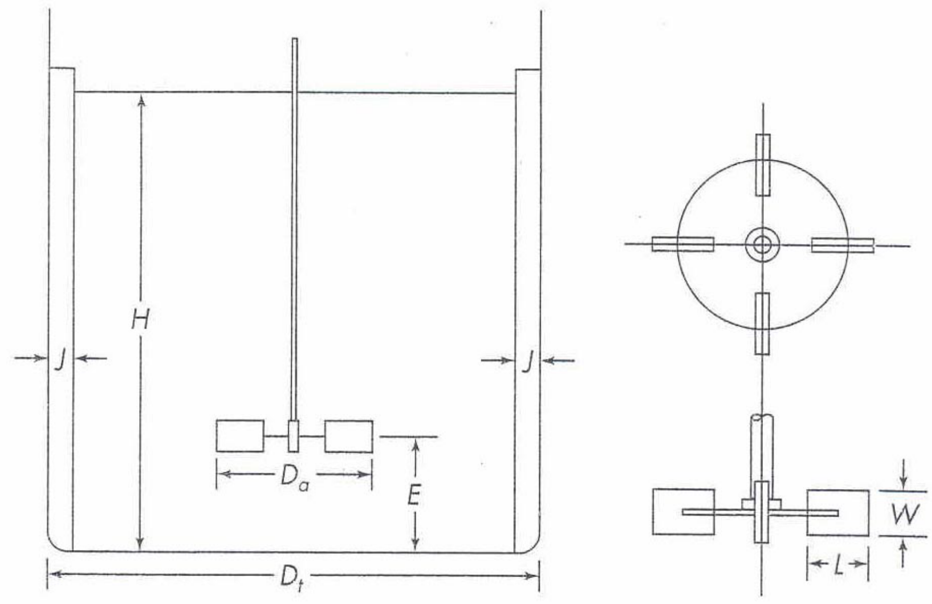

FIGURE 9.3
Measurements of turbine. (*After Rushton et al.*[45])

Figure 5: Nomenclature of agitated vessel parameters with a turbine impeller.

4.2 Basic assumption

- A turbine or propeller agitator is a pump impeller operating without a casing
- The governing relations for turbine are similar to those of centrifugal pumps

4.3 Impeller and fluid velocity

- Considering the diagram of a straight blade turbine
  - u : velocity of the blade tips
  - For the liquid leaving the blade tip
    - V_u : tangential velocity
    - V_r : radial velocity
    - V : overall velocity
- Tangential velocity of the liquid is some fraction of the blade tip velocity

$$V_u = ku$$

- Linear velocity of the impeller is related to its angular velocity by

$$u = \pi D_a n \tag{1}$$

- This gives

$$V_u = k\pi D_a n$$

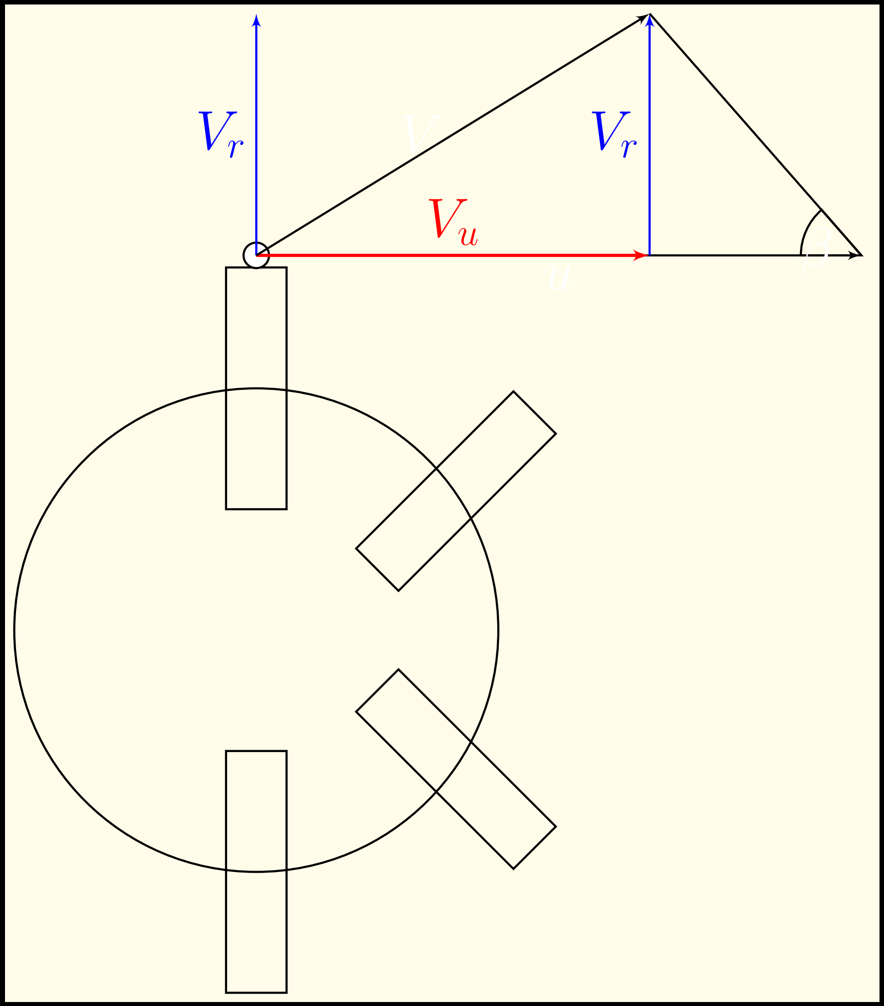

Figure 6: Velocity vector at the tip of turbine impeller blade

4.4 Volumetric flow rate through the impeller

Flow rate of fluid through the impeller is given by

$$q = V_r A_p \tag{2}$$

with A_p is the area swept by the impeller.

$$A_p = \pi D_a W \tag{3}$$

From the trigonometry of the velocity vectors

$$\begin{aligned} V_r &= (u - V_u)\tan\beta \\ &= (\pi D_a n - k\pi D_a n)\tan\beta \\ &= \pi D_a n(1-k)\tan\beta \end{aligned}$$

The volumetric flow of the liquid through the impeller an be obtained as

$$\begin{aligned} q &= V_r A_p \\ &= (\pi D_a n(1-k)\tan\beta)(\pi D_a W) \\ &= \pi^2 D_a^2 n W(1-k)\tan\beta \end{aligned}$$

4.5 Velocity profile of the liquid

- flowing radially from the blade
- velocity is maximum in the plane of the middle of the blade and is much smaller at the edges
- the velocity pattern changes with distance from the impeller tip

4.6 Effective liquid flow rate at the tip of blade

The radial velocity is not constant over the width of the impeller. So q is taken as

$$q = K\pi^2 D_a^2 nW(1-k)\tan\beta \tag{4}$$

4.7 Flow number

With K, k and β approximately constant

$$q \propto nD_a^2W \tag{5}$$

For geometrically similar propellers $W \propto D_a$. So we have

$$q \propto nD_a^3 \tag{6}$$

So we get

$$\frac{q}{nD_a^3} = constant \tag{7}$$

The above ratio which is a constant is known as the flow number and is denoted by N_Q.

4.8 Flow number of different impellers

The flow number depends on the impeller type and is constant for a particular type of impeller. Table 1 shows flow number of different impellers.

Table 1: Flow numbers for different impellers

Type	N_Q
Marine propeller (square pitch)	0.5
4-blade 45^o turbine $\left(\frac{W}{D_a} = \frac{1}{6}\right)$	0.87
Disk turbine	1.3
HE-3 high efficiency impeller	0.47

4.9 Effective liquid flow rate due to entrainment

The discharge flow from the tip of the blade is given by

$$q = N_Q n D_a^3 \tag{8}$$

- The above equation gives the discharge flow from the tip of the impeller
- However, the high velocity jet entrains some liquid from the bulk
- This reduces the velocity but increases the flow rate

For flat-blade turbines, the total flow is estimated experimentally

- by average circulation time for particles or
- by dissolved tracers

and was shown to be

$$q = 0.92 n D_a^3 \left(\frac{D_t}{D_a} \right) \tag{9}$$

This relation is applicable for $\frac{D_t}{D_a}$ between 2 and 4. For a typical ratio $\frac{D_t}{D_a} = 3$

$$q = 2.76 n D_a^3 \tag{10}$$

which is 2.1 times the flow rate from the tip of the blades.

4.10 Power number

- When the flow is turbulent, the power requirement can be estimated from

$$\begin{aligned} P &= \text{flow produced by the impeller} \\ &\quad \times \text{kinetic energy per unit volume of fluid} \end{aligned}$$

$$P = q \times E_k$$

- The kinetic energy is given by

$$E_k = \frac{\rho V^2}{2} \quad (11)$$

- V is smaller that u and if we take V as a fraction of u i.e.

$$V = \alpha u \quad (12)$$

- With $q = N_Q n D_a^3$, the power requirement can be calculated.

$$\begin{aligned} P &= qE_k \\ &= N_Q n D_a^3 \frac{\rho V^2}{2} \\ &= N_Q n D_a^3 \frac{\rho(\alpha \pi n D_a)^2}{2} \\ &= \frac{N_Q n^3 D_a^5 \rho \alpha^2 \pi^2}{2} \end{aligned}$$

- By rearranging to get the constants on one side

$$\frac{P}{n^3 D_a^5 \rho} = \frac{\alpha^2 \pi^2}{2} N_Q \tag{13}$$

- The ratio which is dimensionless is called the power number and is denoted by

$$N_p = \frac{P}{n^3 D_a^5 \rho} \tag{14}$$

4.11 Use of power number

The above relation can be used when N_Q and α are known. For example for a six-blade turbine $N_Q = 1.3$ and if $\alpha = 0.95$, then we get $N_p = 5.8$. Now from the power number, the power requirement of an impeller can be calculated for am impeller with a given diameter rotating at a given speed and for a given liquid.

4.12 Power consumption

Power consumption of an impeller depend on

1. Measurements of the tank and impeller
2. Baffles
   - Baffled: number and arrangements of baffles
   - Unbaffled
3. Liquid properties
   - viscosity
   - density
4. Speed of the impeller
5. Gravitational acceleration

4.13 Shape factors

Measurements are converted into the shape factors which are obtained by dividing the measurements by one of them. Standard shape factors for turbines are

$$\frac{D_a}{D_t} = \frac{1}{3} \qquad \frac{H}{D_t} = 1 \qquad \frac{J}{D_t} = \frac{1}{12}$$
$$\frac{E}{D_t} = \frac{1}{3} \qquad \frac{W}{D_a} = \frac{1}{5} \qquad \frac{L}{D_a} = \frac{1}{4}$$

4.14 Power correlations

- If the shape factors are ignored, then P is a function

$$P = \Psi\left(n, D_a, \mu, \rho, g\right) \tag{15}$$

- By dimensional analysis

$$\frac{P}{n^3 D_a^5 \rho} = \Psi\left(\frac{n D_a^2 \rho}{\mu}, \frac{n^2 D_a}{g}\right) \tag{16}$$

- By taking the shape factors into account

$$\frac{P}{n^3 D_a^5 \rho} = \Psi\left(\frac{n D_a^2 \rho}{\mu}, \frac{n^2 D_a}{g}, S_1, S_2, \cdots, S_i\right) \tag{17}$$

- The dimensionless numbers here are as follows

$$\frac{P}{n^3 D_a^5 \rho} = N_p : Power\ number$$

$$\frac{n D_a^2 \rho}{\mu} = Re : Reynolds\ number$$

$$\frac{n^2 D_a}{g} = Fr : Froude\ number$$

$$N_P = \Psi\left(Re, Fr, S_1, S_2, \cdots, S_i\right) \tag{18}$$

4.15 Reynolds number

Determines the flow type

- $Re < 10$: flow is laminar
- $Re > 10000$: flow is turbulent
- $10 < Re < 10000$: transition region

4.16 Froude number

- appears in fluid dynamic situations where there is significant wave motion
- important for ship design
- not important when baffles are used or $Re < 300$
- unbaffled vessels are rarely used at high Re

Froude number is not included in the correlations for power calculations for mixing equipments.

4.17 Power number as a function of Re

The relations between N_p and Re are expressed graphically and are available for different impellers. An example of such correlation is shown here.

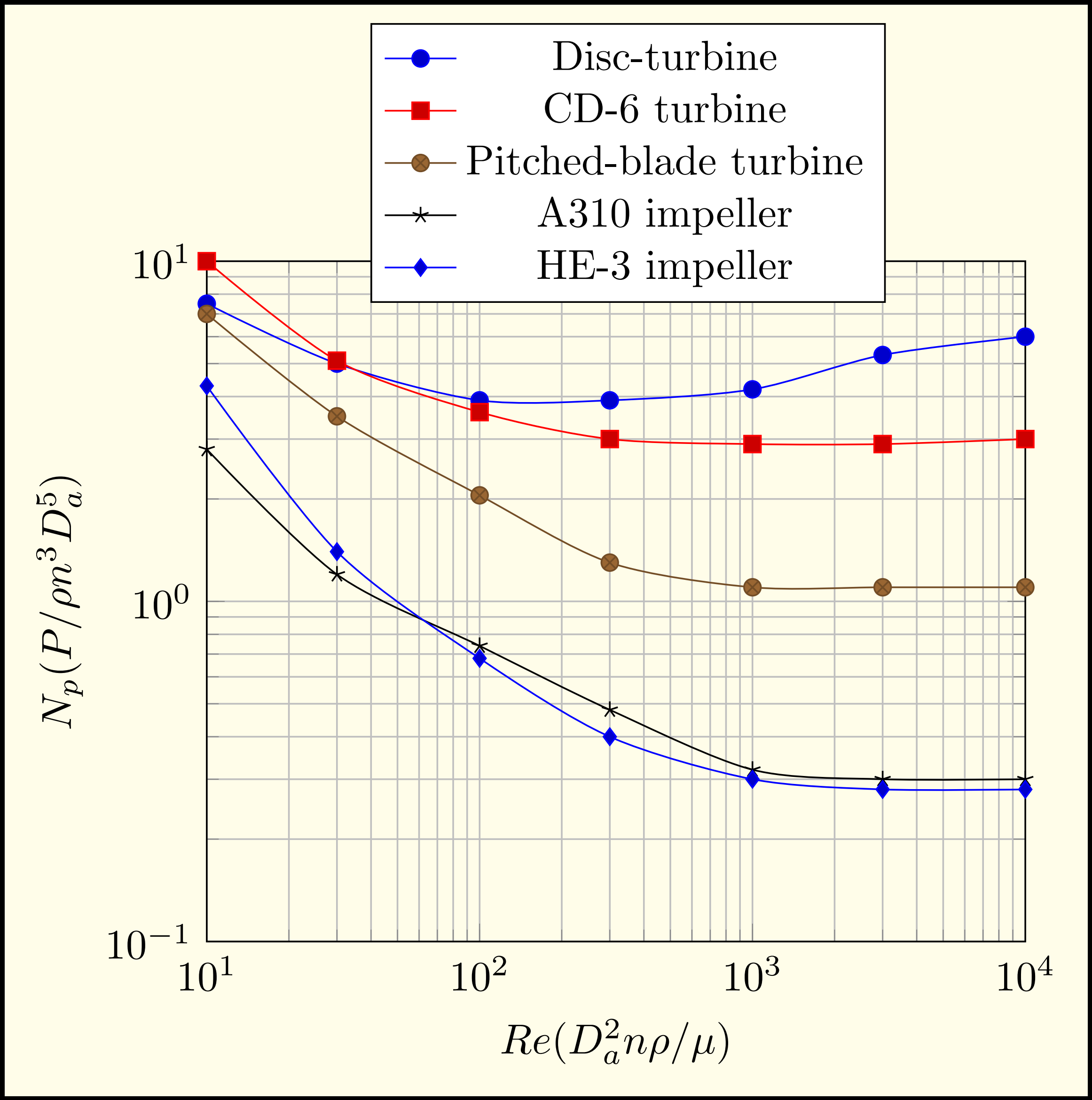

Figure 7: Power number versus Reynold's number for turbines and high efficiency impellers. Redrawn from MacCabe and Smith (2007)

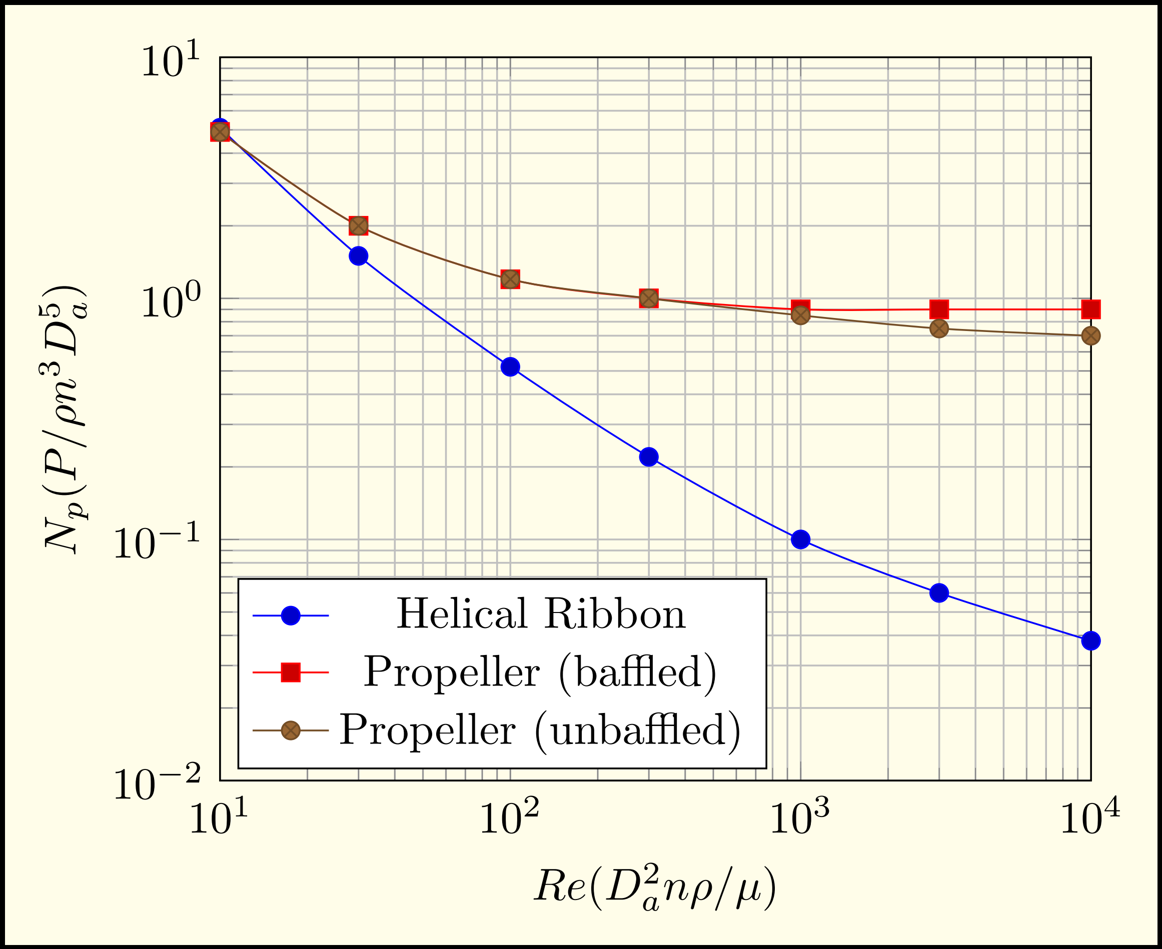

Figure 8: Power number versus Reynold's number for marine propellers (pitch 1.5:1) and helical ribbons. Redrawn from MacCabe and Smith (2007)

4.18 N_p versus Re correlations

Observations from the N_p versus Re correlations:

- At high Re the curves level off.
- The pitched turbine with 4-blades set an angle 45^o
  - draws 0% as much power as standard turbine at low Re
  - draws only about 20% at high Re
- $A301$ and $HE-3$ high efficiency turbines have much lower N_p
- For $Re > 10^4$, N_p remains constant
- For $Re < 10$, N_p varies inversely with Re

For other types of impellers, some observations are as follows:

- Propeller:
  - At $Re \geq 10^4$, N_p is 50% greater in a baffled tank than an

unbaffled tank.
   - At low Re there is no difference.
- Helical ribbon:
   - Baffles are not used
   - N_p decreases rapidly with Re
   - Commonly used at low Re
- Anchor agitator:
   - N_p is slightly greater than helical impellers over the entire range of Re

5 Calculation of power consumption

- From the definition of power number

$$N_p = \frac{P}{n^3 D_a^5 \rho} \tag{19}$$

- we get

$$P = N_p n^3 D_a^5 \rho \tag{20}$$

- If we know the power number of an impeller, we can determine its power requirement. Now N_p can be obtained if Re is known.
  - For $Re < 10$
    - N_p versus Re curve is a straight line with slope -1

$$N_p = \frac{K_L}{Re} = \frac{K_L \mu}{n D_a^2 \rho} \tag{21}$$

- This gives

$$P = \frac{K_L}{Re} n^3 D_a^5 \rho$$
$$= K_L n^2 D_a^3 \mu$$

- Viscosity is a factor, density is not
- Relation is the same for baffled and unbaffled

○ For $Re > 10^4$

- N_p is a constant $N_p \neq f(Re)$

$$N_p = K_T$$

$$P = K_T n^3 D_a^5 \rho$$

- Viscosity is not a factor, density is

Table 2: K_L and K_T for different impellers

Type	K_L	K_T
Propeller, 3 blades		
Pitch 1.0	41	0.32
Pitch 1.5	48	0.87
Turbine		
6-blade disk $(S_3 = 0.25, S_4 = 0.2)$	65	5.75
6-pitched blade $(45^o, S_4 = 0.2)$	-	1.63
4-pitched blade $(45^o, S_4 = 0.2)$	44.5	1.27
Flat paddle, 2-blades $(S_4 = 0.2)$	36.5	1.70
HE-3 impeller	43	0.28
Helical ribbon	52	-
Anchor	300	0.35

6 Blending and mixing

- mixing is much more difficult operation to study than agitation
- velocity pattern of fluid in an agitated vessel is complex but reasonably definite and reproducible
- power consumption for agitation is readily measured
- the results of mixing studies ate seldom highly reproducible
- criteria for good mixing
  - visual - color change, uniformity
  - rate of decay concentration, temperature gradient

6.1 Blending in small process vessels

- Miscible liquids are blended in small process vessels by
- propellers, turbines or high efficiency impellers
- the impellers are usually centrally mounted
- well agitated and rapid mixing

6.2 Blending in large storage tanks

- side entering propellers or jet mixers are used
- idle most of the time
- on during filling

6.3 Flow model for mixing in process vessels

- impeller produces a high velocity stream
- close to the impeller liquid is well mixed
- liquid moves along the wall
  - some radial mixing
  - little mixing in the direction of flow
- the fluid completes a circulation loop and returns to the eye of the impeller
- complete mixing is achieved if the content of the tank is circulated around 5 times
- mixing time can be predicted from the flow correlation

7 Mixing time

For a six-blade turbine

$$q_T = 0.92 n D_a^3 \frac{D_t}{D_a} \tag{22}$$

Now

$$\begin{aligned} t_T &= 5 \times \frac{V}{q_T} \\ &= 5 \times \frac{\pi D_t^2 H}{4} \frac{1}{0.92 n D_a^2 D_t} \end{aligned}$$

So we get

$$nt_T \left(\frac{D_a}{D_t}\right)^2 \left(\frac{D_t}{H}\right) = \frac{5\pi}{4 \times 0.92}$$
$$= 4.3 = constant$$

- for a given tank and impeller or geometrically similar systems, the mixing time is inversely proportional to the stirrer speed provided Re is large.
- mixing time is constant at high Re
- mixing time is appreciable greater when $Re : 10 - 1000$

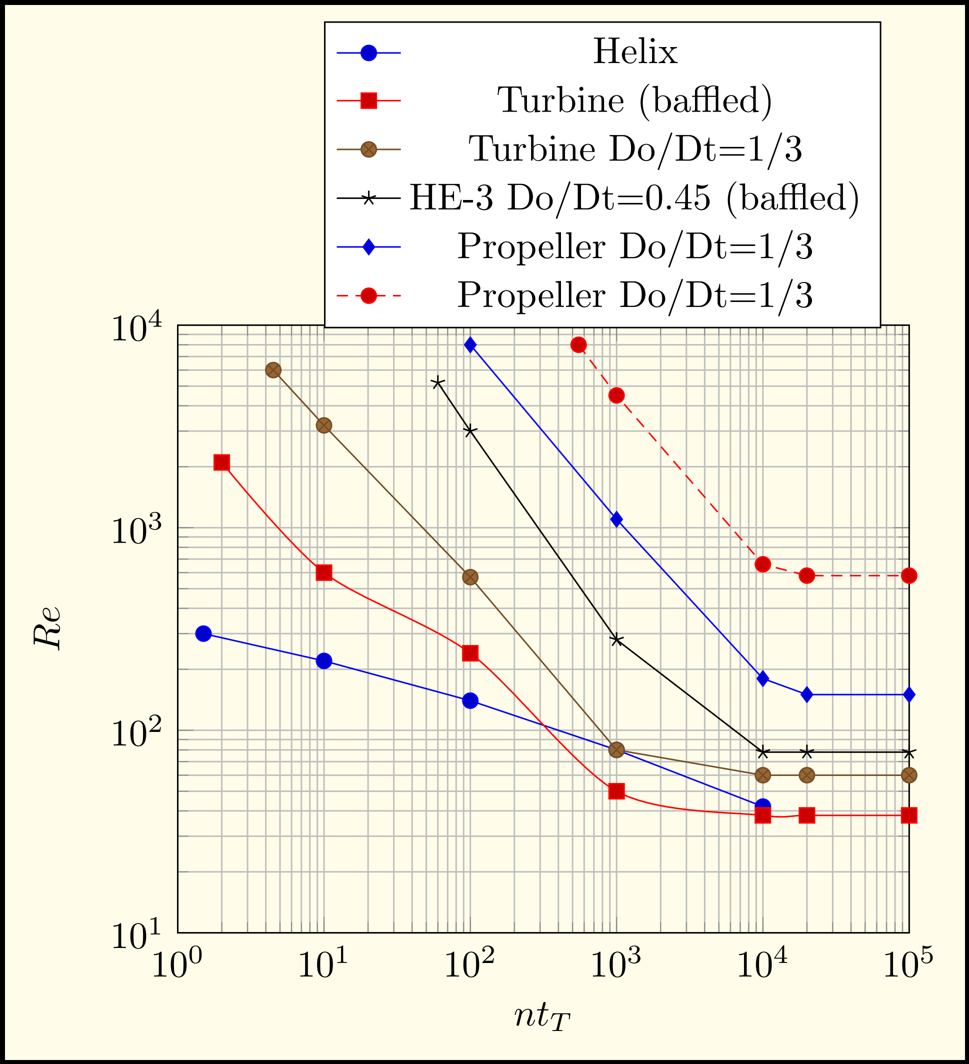

Figure 9: Mixing time for agitated vessels. Redrawn from MacCabe and Smith (2007)

8 Workbook: Power requirement for agitation

A disk turbine with six flat blades is installed centrally in a vertical baffled tank $2\ m$ in diameter. The turbine is $0.67\ m$ in diameter and is positioned $0.67\ m$ above the bottom of the tank. The turbine blades are $134\ mm$ wide. The tank is filled to a depth of $2\ m$ with an aqueous solution of 50 percent $NaOH$ at 65^oC, which has a viscosity of $12\ cP$ and a density of $1500\ kg/m^3$. The turbine impeller turns at $90\ r/min$. What power will be required?

Solution:

- You will have to calculate the power required for the process. Now required power is given by

$$P = N_p n^3 D_a^5 \rho \qquad (23)$$

- Where the power number N_p is a function of Re. Depending on Re, for laminar flow and turbulent flow we get the following relations

$$Laminar\ flow\ : P = K_L n^2 D_a^3 \mu \tag{24}$$
$$Turbulent\ flow\ : P = K_T n^3 D_a^5 \rho \tag{25}$$

- So the strategy will be as follows: Find Re and depending on Re, choose the appropriate equation.
- Then for the impeller under consideration, find the constant. The rest is simple calculation.

9 Workbook: Mixing time requirement

An agitated vessel $6\ ft(1.83\ m)$ in diameter contains a six blade straight blade turbine $2\ ft(0.61\ m)$ in diameter, set one impeller diameter above the vessel floor, and rotating at $80\ r/min$. It is proposed to use this vessel for neutralizing a dilute aqueous solution of $NaOH$ at 70^oF with a stoichiometrically equivalent quantity of concentrated nitric acid (HNO_3). The final depth of liquid in the vessel is to be $6\ ft(1.83\ m)$. Assuming that all the acid is added to the vessel at one time, how long will it take for the neutralization to be complete? Solution:

- Graphical data are available as nt_T as a function of Re.
- If you can find the Re and know the type of impeller, calculate Re to find nt_T which will directly give you t_T for a known n

- So do it!

10 Agitator scale-up

10.1 Need for scale-up

- Experimental data are often available for a laboratory scale or pilot plant system.
- The main design objective is to design a industry scale system.
- To achieve this, scale-up in design calculations are required.

10.2 Scale-up methods

Scale-up process can be approached with diverse requirements:

- Geometric similarity is important and simple to achieve.
- Kinematic similarity defined in terms of ratios of velocities or times, may be required.
- Dynamic similarity requires fixed ratios of viscous, inertial or gravitational forces.

Even if geometric similarity is achieved, dynamic and kinematic similarity cannot be obtained at the same time.

10.3 Scale-up requirements

The objective of a scale-up operation can be different

- Equal liquid motion: the liquid motion i.e. corresponding velocities may be required to keep approximately the same. For example, in blending it may be required to maintain the same motion.
- Equal suspension of solids: the level of suspension may be needed to keep the same.
- Equal rate of mass transfer: where mass transfer takes place between phases it may be required to keep the rate to be the same. This require equal power per unit volume.

10.4 Scale-up procedure

- Step 1: Calculate the scale-up ratio, R.
  - R is defined as the cube root the volume ratio for the final and initial conditions.
  - With D_{a_1}, D_{t_1} and so on denoting the initial values and D_{a_2}, D_{t_2} and so on denoting the final or scaled-up values and assuming that the original vessel is a standard cylinder with properly maintained ratios, the volume V_1 is

$$V_1 = \left(\frac{\pi D_{t_1}^2}{4}\right) H_1 = \frac{\pi D_{t_1}^3}{4} \tag{26}$$

◦ Then the ratio of the volumes is

$$\frac{V_2}{V_1} = \frac{\pi D_{t_2}^3/4}{\pi D_{t_1}^3/4} = \frac{D_{t_2}^3}{D_{t_1}^3} = \frac{D_{a_2}^3}{D_{a_1}^3} \tag{27}$$

◦ R is then defined as

$$R = \left(\frac{V_2}{V_1}\right)^{1/3} = \frac{D_{a_2}}{D_{a_1}} \tag{28}$$

- Step 2: Using R, apply it to all the dimensions to calculate the new values e.g.

$$D_{t_2} = RD_{t_1}, H_2 = RH_1 \tag{29}$$

- Step 3: The scale-up rule is then applied to determine the agitator speed for the duplicate system with

$$\frac{n_2}{n_1} = \left(\frac{1}{R}\right)^{\eta} \tag{30}$$

where

$$\eta = 1 \ : \ for\ equal\ liquid\ motion$$
$$\eta = \frac{3}{4} \ : \ for\ equal\ suspension\ of\ solids$$
$$\eta = \frac{2}{3} \ : \ for\ equal\ rate\ of\ mass\ transfer$$

- Step 4: Knowing n_2, the required power P_2 can be calculated.

10.5 Scale-up case for turbulent flow with constant n

- If the given conditions require constant speed, for turbulent flow, we get

$$P = K_T n^3 D_a^5 \rho \tag{31}$$

- with $V \propto D_a^3$, we get

$$\frac{P}{V} = c n^3 D_a^2 \rho \tag{32}$$

- This gives

$$\frac{P_2/V_2}{P_1/V_1} = \frac{n_2^3 D_{a_2}^2}{n_1^3 D_{a_1}^2} = \left(\frac{n_2}{n_1}\right)^3 \left(\frac{D_{a_2}}{D_{a_1}}\right)^2 \tag{33}$$

- If $n_2 = n_1$, the ratio of power requirement per unit volume becomes

$$\frac{P_2/V_2}{P_1/V_1} = \left(\frac{D_{a_2}}{D_{a_1}}\right)^2 \tag{34}$$

- On the other hand, if the power requirement per unit volume has to remain the same, we get

$$\frac{n_2}{n_1} = \left(\frac{D_{a_1}}{D_{a_2}}\right)^{2/3} \tag{35}$$

- Also from the relation $n_2 t_{T_2} = n_1 t_{T_1}$, we get

$$\frac{t_{T_2}}{t_{T_1}} = \left(\frac{D_{a_2}}{D_{a_1}}\right)^{2/3} \tag{36}$$

11 Workbook: Scale-up in mixing equipment design

A pilot-plant vessel $0.3m$ in diameter is agitated by a six blade turbine impeller $0.1m$ in diameter. When the impeller Reynolds number is 10^4, the blending time of two miscible liquids is found to be $15s$. The power required is $0.4kW/m^3$ of liquid. (a) What power input will be required to give the same blending time in a vessel $1.8m$ in diameter? (b) What would be the blending time in the $1.8m$ vessel if the power input per unit volume were the same as in the pilot plant vessel?

Solution:

- For part (a), the blending time remains the same which implies the same speed of the impeller.
- For part (b), power input per unit volume remains the same.

- You got direct relations for both of the cases.

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