# PROC 5071: Process Equipment Design I

Log Mean Temperature Difference (LMTD)

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# 1 Why log mean?

- We get a general expression for heat transfer in a heat exchanger as

$$Q = UA\Delta T_m \tag{1}$$

- For $\Delta T_m$ a logarithmic mean temperature is used which is commonly known as the Log Mean Temperature Difference or LMTD.
- Question arises "Why log mean?"

# 2 Mathematical formulation

- Let's look at the expression for LMTD

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}} \tag{2}$$

- Here, $\Delta T_1$ and $\Delta T_2$ are the temperature differences between the hot and cold fluid at the two ends and $\Delta T_{lm}$ is the LMTD.
- Figure 1 shows possible temperature profiles of

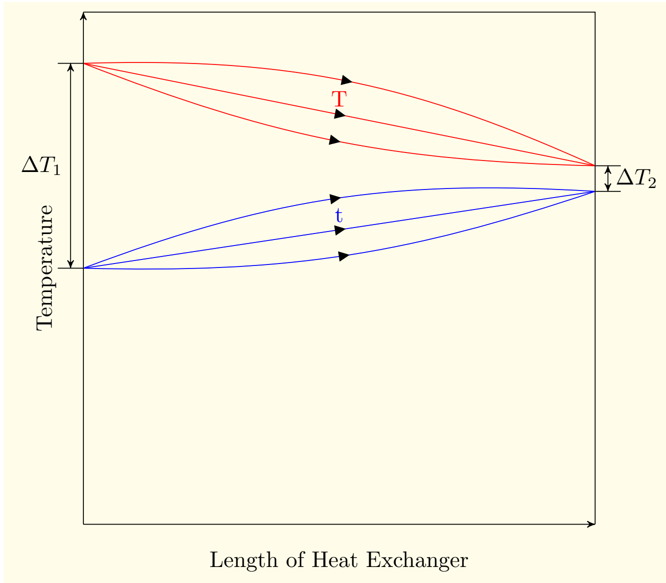


Figure 1: Possible temperature profiles in a parallel flow heat exchanger.

the hot and cold fluids in a heat exchanger.

- Eq. 2 shows that the $\Delta T_{lm}$ depends only on $\Delta T_1$ and $\Delta T_2$.
- Then, does it matter how the temperatures of the two fluids change within the heat exchanger?
- What are the assumptions in defining the LMTD?

## 2.1 Basic heat transfer equations

- In a heat exchanger, the driving force is the temperature difference between the hot and the cold fluid, $\Delta T = T - t$

- Here, we denote the temperatures of the hot and cold fluid by $T$ and $t$, respectively.
- Along the length of the heat exchanger $T$ and $t$ vary significantly and so does the $\Delta T$
- Consequently, the heat flux also varies along the length.
- For a differential area $dA$, the rate of heat flow $dq$ can be expressed as

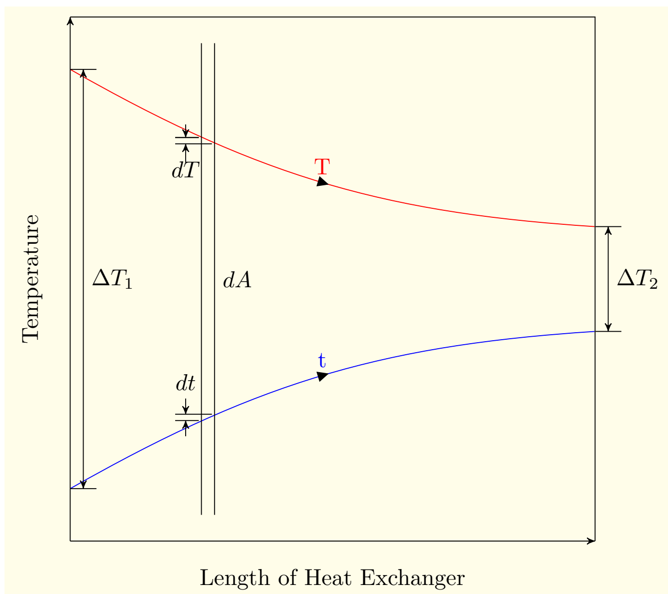


Figure 2: Hot and cold fluid temperature profiles in a heat exchanger.

$$dq = U(dA)\Delta T \tag{3}$$

- Here, $U$ is the local overall heat transfer coefficient.
- For the same differential area the heat transfer

by the hot and cold fluids are given by

$$dq_h = m_h C_{p_h} dT \quad (4)$$
$$dq_c = m_c C_{p_c} dt \quad (5)$$

- Here $m$ and $C_p$ are the fluid mass flow rate and specific heat capacity, respectively.
- The subscripts $h$ and $c$ are used for the hot and the cold fluid.
- $dT$ and $dt$ are the temperature changes in the hot and cold fluids, respectively, between the inlet and outlet of the differential area.
- To get an expression for heat transfer over the entire area using a mean value of $\Delta T$, some assumptions need to be made.

## 2.2 The assumptions

Four simplifying assumptions are made

1. The specific heats of the hot and cold fluids are constant

2. Flow of the fluids are steady and are either parallel or countercurrent
3. Heat loss is negligible
4. The overall heat transfer coefficient is constant

### 2.3 Implications of the assumptions

- Assumption (3) implies that

$$dq_h = dq_c = dq \tag{6}$$

- Assumptions (1) and (2) imply that both $\frac{dT}{dq_h}$ and $\frac{dt}{dq_c}$ are constant
- Assumption (4) imply that $\frac{d(\Delta T)}{dq}$ is constant

The above imply that $T$, $t$ and $\Delta T$ are linear functions of $q$.

### 2.4 Expression for the LMTD

- If the total heat transfer in a heat exchanger is given by $Q$, as $\Delta T$ varies linearly with $q$, we

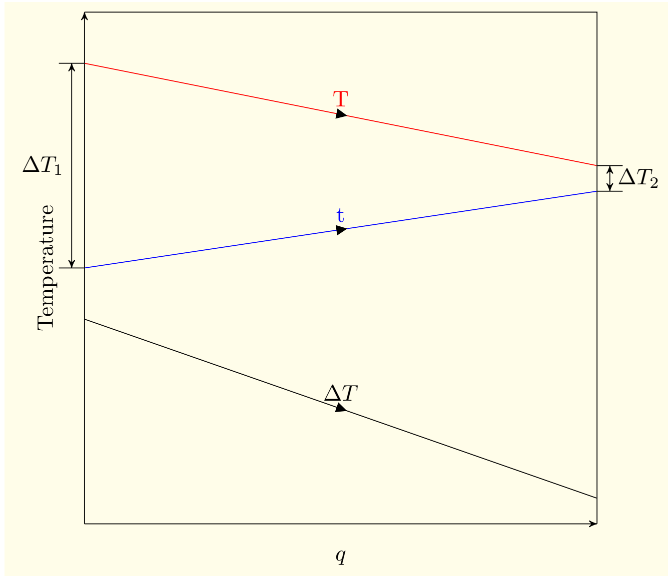


Figure 3: Temperature versus heat flow in a parallel flow heat exchanger.

get

$$\frac{d(\Delta T)}{dq} = \frac{\Delta T_2 - \Delta T_1}{Q} \tag{7}$$

- Using Eq. 3 we get

$$\frac{d(\Delta T)}{U dA \Delta T} = \frac{\Delta T_2 - \Delta T_1}{Q} \tag{8}$$

- A simple rearrangement gives

$$\frac{d(\Delta T)}{\Delta T} = \frac{U(\Delta T_2 - \Delta T_1)}{Q} dA \tag{9}$$

- Now $Q$ is the heat transferred over the entire area, $A$. So integrating over the area between the two ends 1 and 2 gives

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = \int_0^A \frac{U(\Delta T_2 - \Delta T_1)}{Q} dA \tag{10}$$

- Integration and use of the limits will give

$$\ln \frac{\Delta T_2}{\Delta T_1} = \frac{U(\Delta T_2 - \Delta T_1)}{Q} A \tag{11}$$

- So we get

$$Q = UA \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} \tag{12}$$

- Comparing with Eq. 1, we get the expression for the mean temperature, which is known as LMTD

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}} \tag{13}$$

## 2.5 Applicability and limitations

Note that the assumptions mentioned above should be satisfied for the Eq. 16 to be applicable. The LMTD measure is not applicable when

- $\Delta T_1$ and $\Delta T_2$ are equal or nearly equal. In such a case the arithmetic mean can be used.
- $U$ changes significantly
- $\Delta T$ is not a linear function of $q$

## 2.6 Special case with variable $U$

- When $U$ is not constant, if it changes linearly with $\Delta T$ over the entire range of the heat exchanger, a log mean of the entire term $U\Delta T$ can be used
- In that case, one can write

$$Q = A(U\Delta T)_{lm} \tag{14}$$

- with

$$(U\Delta T)_{lm} = \frac{U_2\Delta T_1 - U_1\Delta T_2}{\ln \frac{U_2\Delta T_1}{U_1\Delta T_2}} \qquad (15)$$

- Note that in this equation the multiplication term contains $U$ of one end with $\Delta T$ of the other end
- The derivation is shown in the Appendix section.

# 3 Workbook: LMTD calculation for parallel and countercurrent flow

## 3.1 The problem

Methanol condensate is to be subcooled from $95^oC$ to $50^oC$. Water will be used as the coolant, with a temperature rise from $25^oC$ to $40^oC$. If a double pipe heat exchanger is used, calculate the LMTD for both parallel flow and countercurrent flow arrangement.

## 3.2 Notes and analysis

- The expression for LMTD is given by

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}} \tag{16}$$

- Note that in this expression $1$ and $2$ denote the two ends of the heat exchanger.
- So for parallel and countercurrent flow the definitions of $\Delta T_1$ and $\Delta T_2$ will be different.
- We will denote the inlet end of the hot stream as end $1$. Also we will use $T$ for the hot stream and $t$ for the cold stream.

## 3.3 Parallel flow

**Step 1** : Identify the temperatures.

Using the above notations, for the parallel flow

$$\begin{aligned} T_1 &= \quad {}^oC \\ T_2 &= \quad {}^oC \\ t_1 &= \quad {}^oC \\ t_2 &= 40^oC \end{aligned}$$

**Step 2** : Calculate $\Delta T_1$ and $\Delta T_2$.

$$\begin{aligned} \Delta T_1 &= T_1 - t_1 \\ &= \\ &= \end{aligned}$$

$$\begin{aligned} \Delta T_2 &= T_2 - t_2 \\ &= \\ &= \end{aligned}$$

**Step 3** : Calculate $\Delta T_{lm}$.

$$\begin{aligned} \Delta T_{lm} &= \frac{}{\ln} \\ &= 30.8^oC \end{aligned}$$

## 3.4 Countercurrent flow

- Note that only Step 1 in this approach is different for parallel and countercurrent flow.
- With end $1$ being the inlet for the hot stream, it's the outlet for the cold stream in the countercurrent setting.
- Once $T_1, T_2$ and $t_1, t_2$ are defined, Step 2 and 3 remain the same.

**Step 1** : Identify the temperatures.

Using the notations described earlier, for the countercurrent flow

$$\begin{aligned} T_1 &= \quad {}^oC \\ T_2 &= \quad {}^oC \\ t_1 &= 40^oC \\ t_2 &= \quad {}^oC \end{aligned}$$

**Step 2** : Calculate $\Delta T_1$ and $\Delta T_2$.

$$
\begin{aligned}
\Delta T_1 &= T_1 - t_1 \\
&= \\
&=
\end{aligned}
$$

$$
\begin{aligned}
\Delta T_2 &= T_2 - t_2 \\
&= \\
&= \quad {}^oC
\end{aligned}
$$

**Step 3** : Calculate $\Delta T_{lm}$.

$$
\begin{aligned}
\Delta T_{lm} &= \frac{\qquad\qquad {}^oC}{\ln} \\
&= 38^oC
\end{aligned}
$$

### 3.5 Comment

- Note that the LMTD is significantly higher for the countercurrent flow than that for the parallel flow.
- For a given set of temperatures, that is always the case.

# 4 LMTD calculation for multi-pass shell and tube heat exchangers

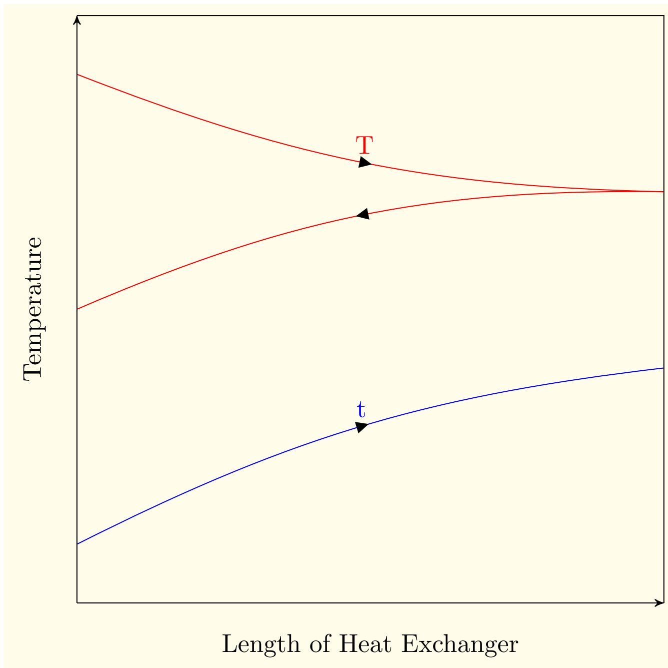


Figure 4: Temperature profiles in a 1-2 shell and tube heat exchanger.

- For multipass heat exchangers, the temperature profiles become complex.
- The concept of inlet end and outlet end become invalid.
- For multipass heat exchangers, an approximate method is used to calculate the mean temperature difference.

### 4.1 Calculation steps

**Step 1** : LMTD ($\Delta_{lm}$) is calculated assuming a single pass countercurrent flow.

**Step 2** : A correction factor, $F_T$, is calculated base on the temperatures of the hot and cold stream and the type of heat exchanger.

**Step 3** : The corrected mean temperatures is calculated as.

$$\Delta T_m = F_T \times \Delta_{lm} \tag{17}$$

## 5 Workbook: LMTD calculation for a 1-2 shell and tube heat exchanger

### 5.1 The problem

Methanol condensate is to be subcooled from $95^oC$ to $50^oC$. Water will be used as the coolant, with a temperature rise from $25^oC$ to $40^oC$. If a 1-2 shell and tube heat exchanger is used, calculate the corrected mean temperature.

## 5.2 Notes and analysis

- We will use the notations $T$ and $t$ for the hot and cold fluid temperature, respectively.
- For the ends, $1$ is used for the inlet of the hot stream.
- For countercurrent, end $2$ is the inlet for the cold fluid.

## 5.3 Calculation steps

**Step 1** : Calculate LMTD ($\Delta_{lm}$) assuming a single pass countercurrent flow.

**1.1** : Identify the temperatures.

Using the above mentioned notations, for the countercurrent flow

$$
\begin{aligned}
T_1 &= \quad {}^oC \\
T_2 &= \quad {}^oC \\
t_1 &= 40^oC \\
t_2 &= \quad {}^oC
\end{aligned}
$$

**1.2** : Calculate $\Delta T_1$ and $\Delta T_2$.

$$
\begin{aligned}
\Delta T_1 &= T_1 - t_1 \\
&= \\
&=
\end{aligned}
$$

$$
\begin{aligned}
\Delta T_2 &= T_2 - t_2 \\
&= \\
&= \quad {}^oC
\end{aligned}
$$

**1.3** : Calculate $\Delta T_{lm}$.

$$
\begin{aligned}
\Delta T_{lm} &= \frac{\qquad\qquad {}^oC}{\ln} \\
&= 38^oC
\end{aligned}
$$

**Step 2** : Calculate the correction factor, $F_T$.

**2.1** : Calculate the constants $R$ and $S$

$R$ is defined as the ratio of temperature decrease of the hot fluid to the temperature increase of the cold fluid

$$\begin{aligned} R &= \frac{T_1 - T_2}{t_2 - t_1} \\ &= \frac{(\quad - \quad)^oC}{(\quad - \quad)^oC} \\ &= \end{aligned}$$

$S$ is defined as the ratio of the temperature increase of the cold fluid to the difference in the inlet temperature of the two fluids

$$\begin{aligned} S &= \frac{t_2 - t_1}{T_1 - t_2} \\ &= \frac{(\quad - \quad)^oC}{(\quad - \quad)^oC} \\ &= \end{aligned}$$

**2.2** : From the graph find the value of the correction factor.

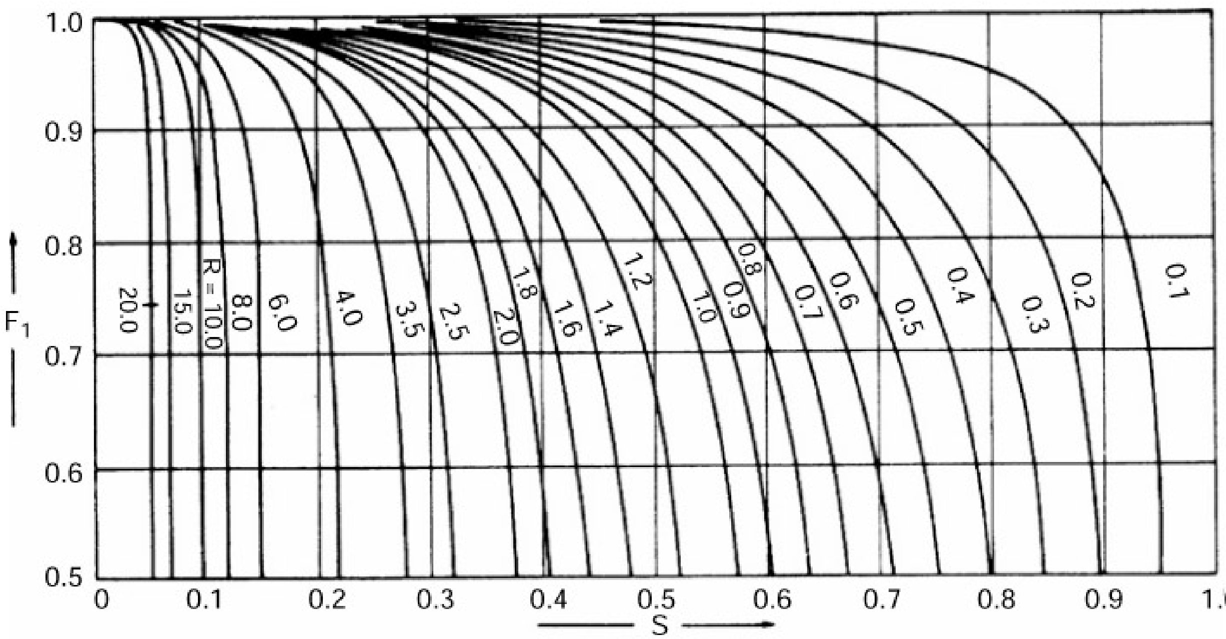


Figure 5: Temperature correction factor for 1-2 shell and tube heat exchangers.

Alternatively, the following equation can be used to calculate $F_T$.

$$F_T = \frac{\sqrt{R^2+1}\ln\left[\frac{1-S}{1-RS}\right]}{(R-1)\ln\left[\frac{2-S\left(R+1-\sqrt{R^2+1}\right)}{2-S\left(R+1+\sqrt{R^2+1}\right)}\right]}$$

**Step 3** : Calculate the corrected mean temperature.

$$
\begin{aligned}
\Delta T_m &= F_T \times \Delta T_{lm} \\
&= \qquad \times 38^o C \\
&= \quad {}^o C
\end{aligned}
$$

# 6 Use of $F_T$ for selecting heat exchanger configuration

- The configuration of the heat exchanger (number of shell and tube passes) is selected to get a desired $F_T$.
- It is desirable to have $F_T > 0.85$.
- $F_T < 0.75$ is generally unacceptable.
- For a given number of shell pass, the value of $F_T$ is not affected significantly on the number of tube passes.
- The more shell passes, the higher is the value of $F_T$.

# Appendix

- This section provides the derivation of the heat transfer equation when $U$ is not constant and changes linearly with $\Delta T$
- WE start with Eq. 8

$$\frac{d(\Delta T)}{UdA\Delta T} = \frac{\Delta T_2 - \Delta T_1}{Q} \tag{18}$$

- We rearrange the equation considering that $U$ is a function of $\Delta T$

$$\frac{d(\Delta T)}{U\Delta T} = \frac{\Delta T_2 - \Delta T_1}{Q} dA \tag{19}$$

- As $U$ changes linearly with $\Delta T$, we can express $U$ as

$$U = a + b\Delta T \tag{20}$$

- As $U_1 = a + b\Delta T_1$ and $U_2 = a + b\Delta T_2$

$$a = \frac{U_2 \Delta T_1 - U_1 \Delta T_2}{\Delta T_2 - \Delta T_1}$$

$$b = \frac{U_2 - U_1}{\Delta T_2 - \Delta T_1}$$

- Integrating the equation between the limits at the two ends

$$\int_{\Delta T_1}^{\Delta T_1} \frac{d(\Delta T)}{(a + b\Delta T)\Delta T} = \int_0^A \frac{U(\Delta T_2 - \Delta T_1)}{Q} dA \tag{21}$$

- We will use the following formula

$$\int \frac{dx}{(a + bx)x} = \frac{1}{a} \ln \frac{x}{a + bx} \tag{22}$$

- Upon integration, we get left hand side (LHS) of the equation as

$$\begin{aligned} LHS &= \frac{1}{a} \ln \frac{\Delta T}{a + b\Delta T}\bigg|_{\Delta T_1}^{\Delta T_1} \\ &= \frac{1}{a}\left[\ln \frac{\Delta T_2}{a + b\Delta T_2} - \ln \frac{\Delta T_1}{a + b\Delta T_1}\right] \\ &= \frac{\Delta T_2 - \Delta T_1}{U_2\Delta T_1 - U_1\Delta T_2} \ln \frac{U_1\Delta T_2}{U_2\Delta T_1} \end{aligned} \tag{23}$$

- So we have

$$\frac{\Delta T_2 - \Delta T_1}{U_2\Delta T_1 - U_1\Delta T_2}\ln\frac{U_1\Delta T_2}{U_2\Delta T_1} = \frac{\Delta T_2 - \Delta T_1}{Q}A \qquad (24)$$

- Upon rearrangement, one get

$$Q = A\frac{U_2\Delta T_1 - U_1\Delta T_2}{\ln\frac{U_2\Delta T_1}{U_1\Delta T_2}} \qquad (25)$$

- Comparing Eq. 25 with Eq. 14, we get the expression of $(U\Delta)_{lm}$ as in Eq. 16.

# References


1. W. L. McCabe, J. C. Smith, P. Harriott. (2005). Unit Operations of Chemical Engineering, 7th Edition, McGraw Hill, New York, USA.
2. G. Towler and R. Sinnott. *Chemical Engineering Design: Principles, Practice and Economics of Plant and Process Design*. Butterworth-Heinemann 2008.