PROC 5071:
Process Equipment Design I

Fluidization

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1 Fluidization

1.1 What is fluidization?

The dictionary definition of fluidize is *'to cause to flow like a fluid'*.

- Fluidization - the process to cause something to behave like a fluid.
- Among the three phases two are fluid.
- Fluidization is to make solids behave like a fluid.

1.2 How is fluidization caused?

When a liquid or a gas is passed through a bed of solid particles

- at a very low velocity of the fluid, the solid particles do not move
- if the fluid velocity is steadily increased, the drag on individual particles increases
- at some point the particles start to move and become suspended in the fluid.

The suspension of the solid in the fluid is called fluidized bed and the process is referred to as fluidization.

1.3 Fluidic behavior of the suspension

- If the bed is tilted, the top surface remains horizontal.
- Objects will float or sink in the bed depending on their density relative to the suspension.
- The fluidized solids can be drained from the bed through pipes and valves.

The fluidity is one of the main advantages of the use of fluidization for handling solids.

1.4 Industrial use of fluidization

- Catalytic reactions:
  - Fluidized catalytic cracking
  - Chlorination of olefins
  - Conversion of methanol to gasoline
- Noncatalytic reactions: The solids act as heat sink or source.
  - Chlorination of hydrocarbons
  - Roasting of ores to release value metals
  - Incineration of biological sludge
- Drying
  - coal, cement, rock, limestone
- Classification
  - Separation of fine particles from coarse

- Heat treatment
  - large objects conveyed through a fluidized bed for heat treatment
- Coating
  - Heated metal part is dipped into fluidized bed of thermoplastic resin. The metal fuses the resin to form a uniform coating.

1.5 Advantages of fluidization

- Fluidic flow is easy to control and automate
- Large surface area gives high heat and mass transfer
- Rapid and rigorous mixing give uniform temperature and concentration

1.6 Disadvantages of fluidization

- For fine particles, bubbling may occur and it is difficult to predict
- Particles may break up
- Particle collision causes erosion of vessel
- Rapid mixing may cause non-uniform residence time for catalytic reactions

1.7 Questions to be answered for a fluidization problem

- What minimum flow rate of fluid should be maintained to cause fluidization?
- How much pressure drop will be caused due to fluidization?
- To what extend will the bed expand at a certain flow rate of fluid?

2 Relevant theories on fluid mechanics

2.1 Flow passed immersed bodies

- You might have been introduced to the concept of frictional loss for flow of fluids inside pipes.
- Similar loss takes place when fluid flow around immersed solid objects.
- flow of fluids outside immersed bodies occurs in many chemical applications such as: flow past spheres in settling, flow through packed beds in drying and filtration, flow past tubes in heat exchangers and so on.

2.2 Drag forces

- When fluid flows over a solid surface, the transfer of momentum perpendicular to the surface results in a tangential shear stress or drag on the smooth surface parallel to the direction of the flow. The force exerted by the fluid on the solid in the direction of the flow is called skin or wall drag.
- In addition to skin drag, if the fluid has to change its direction to pass around a solid body such as sphere, significant additional frictional losses will occur and this is called form drag.
- The tangential stress on the body because of the velocity gradient in the boundary layer is the skin friction.
- Outside the boundary layer, the fluid change direction to pass around the body accelerating near the front then decelerating.

- Thus, an additional force is exerted by the fluid on the body: This is the form drag.
- Separation of the boundary layer occurs and a wake covering the entire rear of the body occurs where large eddies are present and contribute to the form drag.
- Form drag for bodies can be minimized by streamlining the body which forces the separation point toward the rear of the body, reducing the size of the wake.

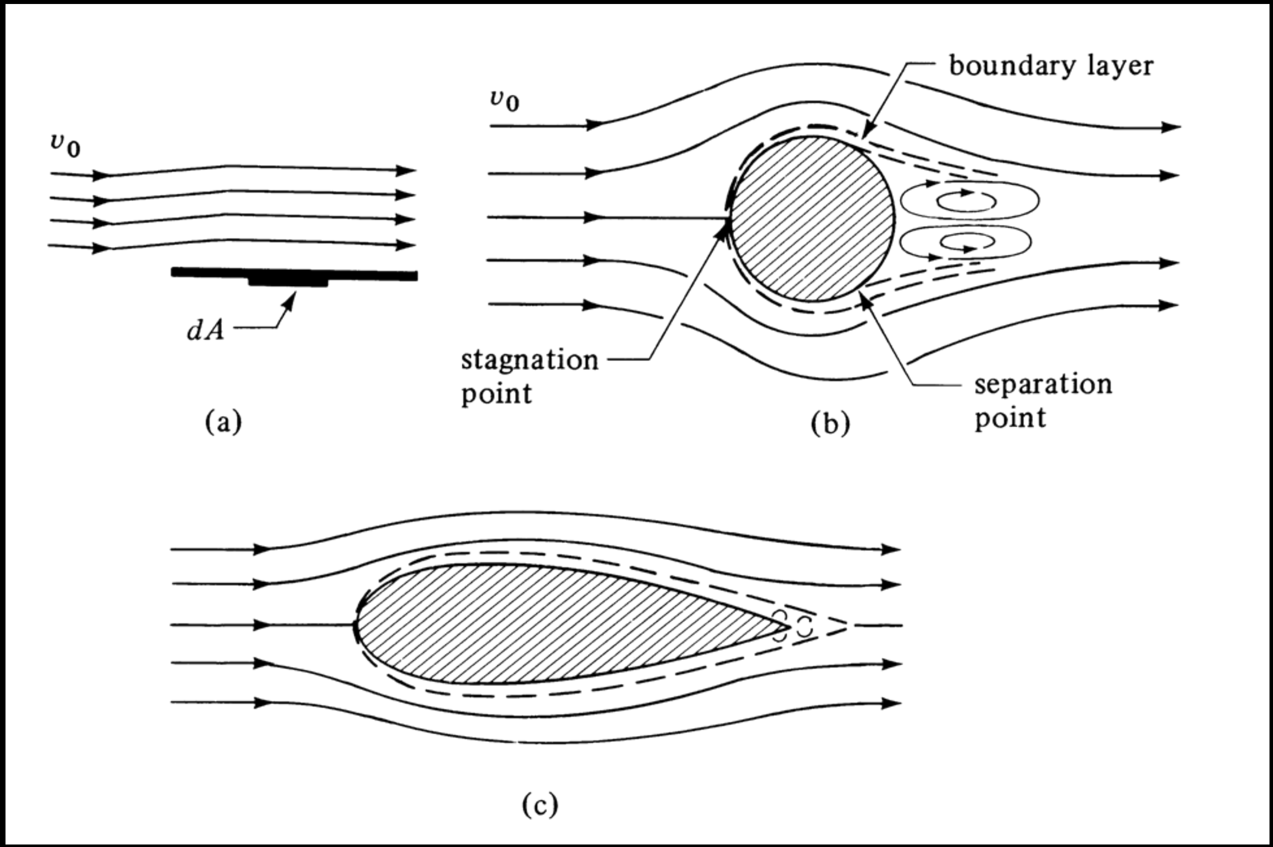


Figure 1: Drag forces: Skin or wall drag, form drag.

2.3 Flow Separation

- Due to its viscosity, fluid very close to the surface of solid slows down and a thin slow moving fluid layer called the boundary layer is formed on the solid surface.
- The flow velocity is zero at the surface at the no-slip boundary condition.
- Inside the boundary layer, flow momentum is quite low since it experiences a strong viscous flow resistance. Therefore, the boundary layer flow is sensitive to the external pressure gradient.
- If the pressure decreases in the direction of the flow, the pressure gradient is said to be favorable. In this case, the pressure force can assist the fluid movement and there is no flow retardation.
- However, if the pressure is increasing in the direction of the flow,

an adverse pressure gradient condition exists.

- In addition to the presence of a strong viscous force, the fluid particles now have to move against the increasing pressure force. Therefore, the fluid particles could be stopped or reversed, causing the neighboring particles to move away from the surface. This phenomenon is called the boundary layer separation.

2.4 Wake

- Consider a fluid particle flows within the boundary layer around the circular cylinder.
- Pressure is a maximum at the stagnation point and gradually decreases along the front half of the cylinder.
- The flow stays attached in this favorable pressure region.
- However, the pressure starts to increase in the rear half and the particle experiences an adverse pressure gradient.
- Consequently, the flow separates from the surface and creating a highly turbulent region behind the cylinder called the wake.
- The pressure inside the wake region remains low as the flow separates and a net pressure force (pressure drag) is produced.

2.5 Coefficient of drag

- For fluids flowing through pipes, a friction factor, defined as the ratio of the shear stress to the product of the velocity head and density, is used.
- For drag force a similar factor, called the drag coefficient, is defined.
- If F_D is the total drag and A_p is the projected area of particle, average drag per unit area is given by F_D/A_p.
- The drag coefficient, C_D, is defined as the ratio of F_D/A_p to the product of fluid density $(\rho)$and the velocity head $(u_0^2/2)$.

$$C_D = \frac{F_D/A_p}{\rho u_0^2/2} \tag{1}$$

2.6 Projected area

- Projected area of a particle is the area of the particle seen by the fluid.

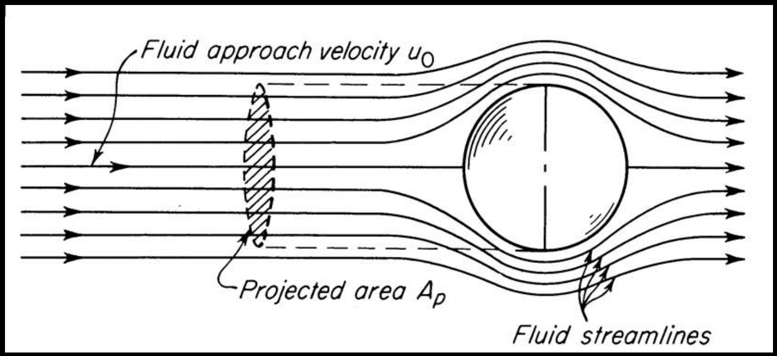
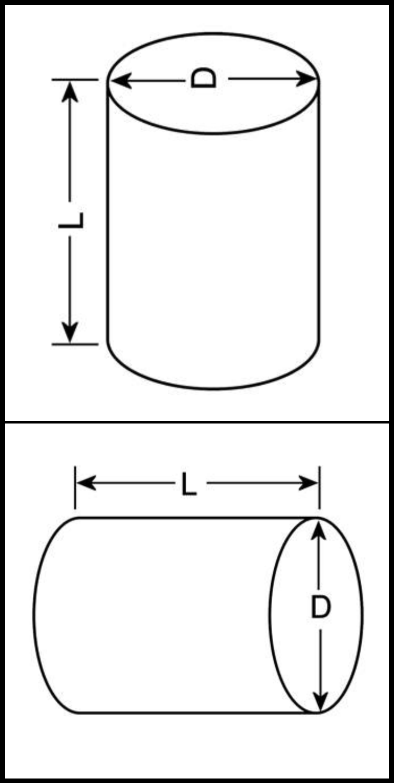


Figure 2: Projected area of a spherical particle.

For a sphere which is symmetric around all directions, fluid approaching it from any direction will see it as a circular disk. So for a sphere, the projected area is

$$A = \pi R^2 = \pi \frac{D^2}{4} \qquad (2)$$



- Axis is perpendicular to flow
  - The projected area is a rectangle
  - $A = LD$
- Axis is parallel to flow
  - The projected area is a circle
  - $A = \pi \frac{D^2}{4}$

2.7 Drag coefficient for typical shapes

- The drag coefficient of a smooth solid in an incompressible fluid depends on the Reynolds number and the necessary shape factors.
- For a given shape

$$C_D = \phi(Re_p) \tag{3}$$

where

$$Re_p = \frac{D_p u_0 \rho}{\mu} \tag{4}$$

- The subscript 0 in u_0 refers to what is called the approach velocity which is the velocity of the fluid far away from the solid. Note that close to the particle acceleration and deceleration of fluid take place.
- The function ϕ is different for different shape and orientation. The

relation should be determined experimentally.

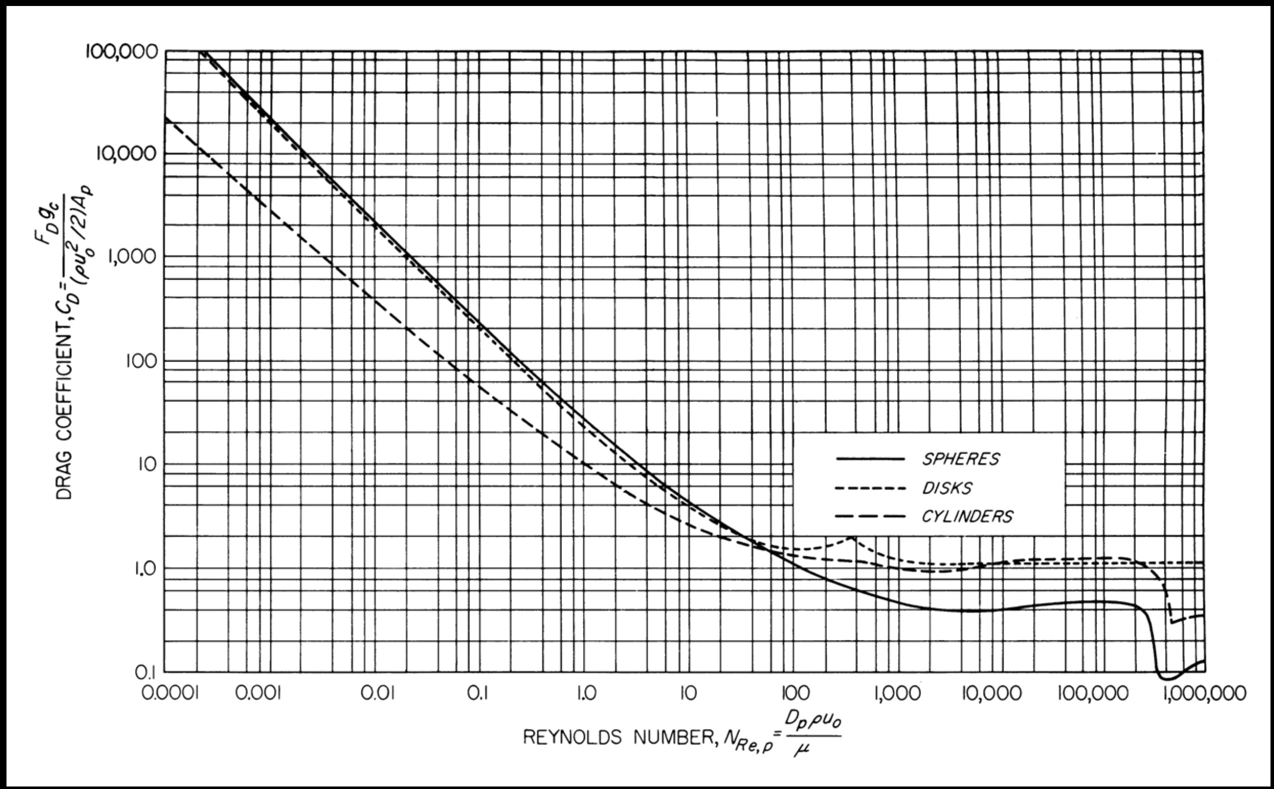


Figure 3: Drag coefficient for spheres, disks and cylinders .

- However, a well-established equation exists for spherical particles at low Re.

- For $Re < 1$, the drag force for spheres can be obtained using the Stokes' law

$$F_D = 3\pi\mu u_0 D_p \tag{5}$$

- The projected area of a sphere is given by

$$A_p = \frac{\pi D_p^2}{4} \tag{6}$$

- These results in the equation for C_D as

$$C_D = \frac{24}{Re} \tag{7}$$

2.8 Flow through solid beds

- The resistance to the flow of a fluid through the voids in a bed of solids is the resultant of the total drag of all particles in the bed.
- The most common method of calculating the Δp through a bed of solids is based on estimates of total drag on the solid boundaries of the tortuous channels through the bed.

3 Fluidization - phenomenon and required conditions

- Fluidization is caused by the flow of a fluid through a bed of particles.
- At low velocity fluid does not impart enough drag to overcome gravity and particles do not move.
- As velocity is increased, the drag force increases and so the pressure drop. However, upto certain velocity the drag force is not enough and the bed remains fixed.
- At high enough velocity fluid drag plus buoyancy overcomes the gravitational force and the bed expands, making the bed fluidized.
- After the onset of fluidization, Δp remains constant; the length of the bed increases with velocity.

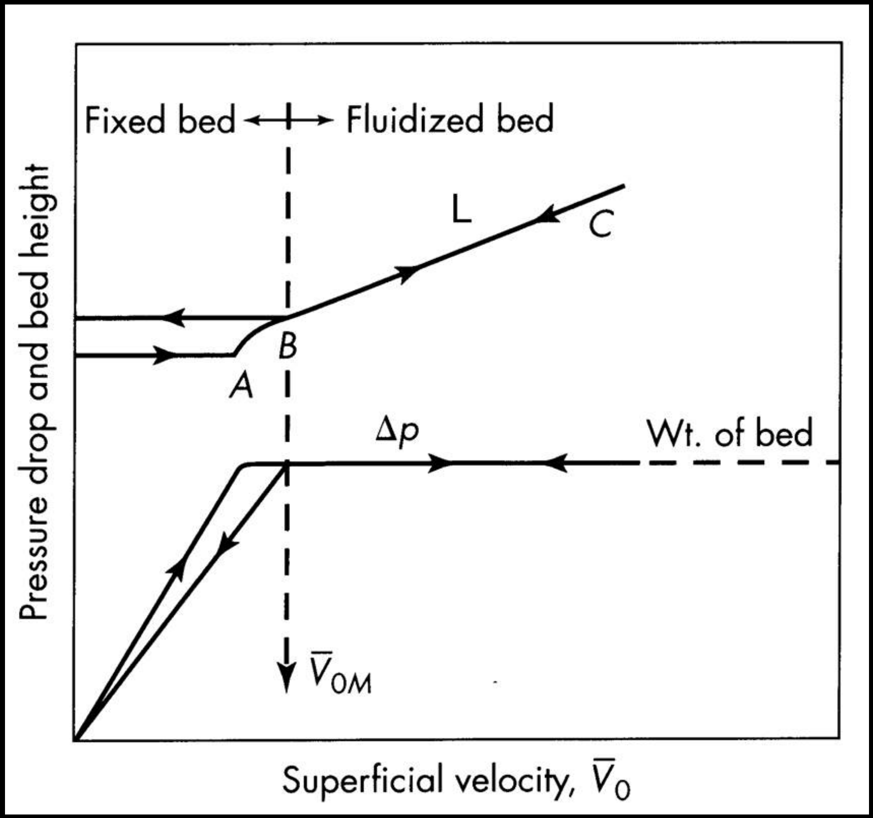


Figure 4: Pressure drop and bed height versus superficial velocity for a bed of solids [Source: [**?**].

4 Mathematical formulations for fluidization

4.1 Pressure drop in fluidized bed

- The pressure drop can be related to other acting forces on the bed by the mechanical energy equation

$$\frac{p_1}{\rho} + gL_1 + \frac{\alpha_1 v_1^2}{2} = \frac{p_2}{\rho} + gL_2 + \frac{\alpha_2 v_2^2}{2} + h_f \quad (8)$$

- Making the following assumptions
  - Potential energy change is negligible
  - $v_1 = v_2$ and $\alpha_1 = \alpha_2$

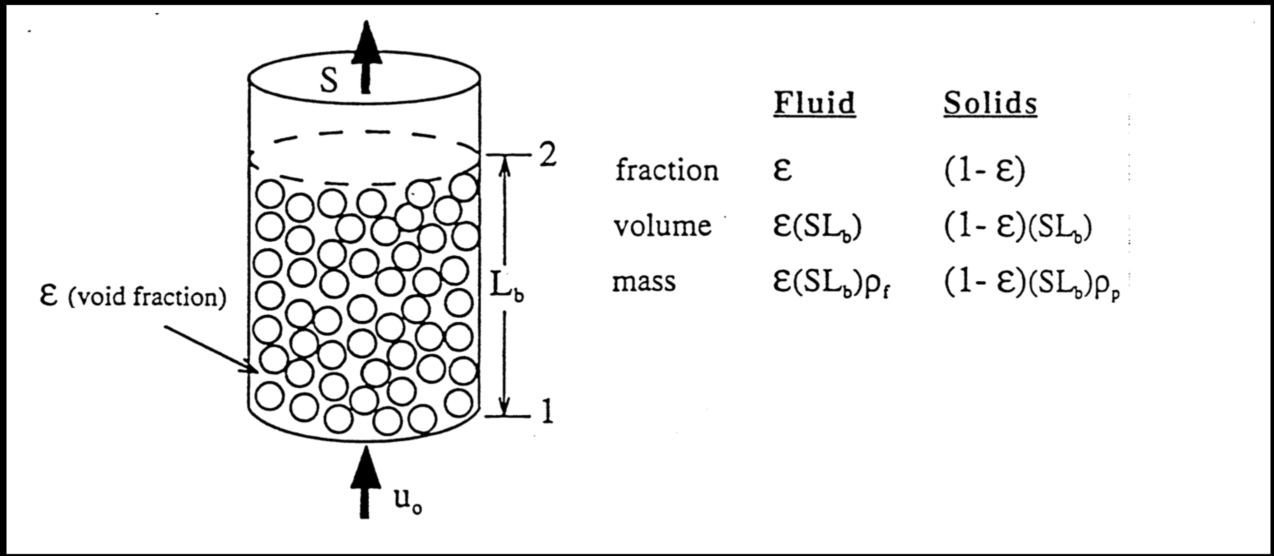


Figure 5: Schematic of a fluidized bed.

We get

$$\frac{p_2 - p_1}{\rho} = h_f \tag{9}$$

- With further assumption that
  - Particles are packed uniformly giving rise to continuous flow channels

◦ Bed can be modeled as bundle of small pipes
we can use the pipe flow relation which give

$$h_f = 4f\left(\frac{L}{D}\right)\frac{\overline{V}^2}{2} \tag{10}$$

- Using this relation

$$\frac{\Delta p}{\rho} = 4f\left(\frac{L}{D}\right)\frac{\overline{V}^2}{2} \tag{11}$$

4.2 Laminar flow

- If the flow is laminar we get

$$f = \frac{16}{Re} \tag{12}$$

- So we get

$$\begin{aligned}\frac{\Delta p}{\rho} &= 4\frac{16}{Re}\left(\frac{L}{D}\right)\frac{\overline{V}^2}{2} \\ &= \frac{32\mu L\overline{V}}{D^2\rho}\end{aligned}$$

- Giving

$$\frac{\Delta p}{L} = \frac{32\mu\overline{V}}{D^2} \tag{13}$$

- The velocity of the fluid in the channels, $\overline{V}$ can be obtained from the superficial velocity (empty tower velocity) by using the mass balance

$$\rho\overline{V}_0 S = \rho\overline{V}\epsilon S$$
$$\overline{V} = \frac{\overline{V}_0}{\epsilon}$$

- Also D should be the hydraulic radius which is given by

$$D_h = 4\times\frac{\text{cross}-\text{sectional area of flow}}{\text{wetted perimeter}}$$
$$= 4\times\frac{\text{volume available for flow}}{\text{wetted surface area}}$$
$$= 4\times\frac{\epsilon LS}{(1-\epsilon)LSa_s}$$

- where a_s is the surface area of particle per unit volume which is given by

$$a_s = \frac{6}{\Phi_s D_p}$$

- With this we get

$$D_h = \frac{2}{3}\frac{\epsilon}{1-\epsilon}\Phi_s D_p$$

- Using the above expressions

$$\frac{\Delta p}{L} = \frac{72\mu \overline{V}_0}{(\Phi_s D_p)^2}\frac{(1-\epsilon)^2}{\epsilon^3} \tag{14}$$

- The above equation does not account for the tortuous path through the bed and L is much longer.
- Experimental data show that a numerical constant of 150 should

replace 72 which gives

$$\frac{\Delta p}{L} = \frac{150\mu \overline{V}_0}{(\Phi_s D_p)^2} \frac{(1-\epsilon)^2}{\epsilon^3} \tag{15}$$

- The above equation is called Kozeny-Carman equation.
- This equation is applicable for flow through beds of particle for Re up to about 1.

4.3 Turbulent flow

- For turbulent flow the basic equation for pressure drop is still valid

$$\frac{\Delta p}{\rho} = 4f\left(\frac{L}{D}\right)\frac{\overline{V}^2}{2} \tag{16}$$

- However, the above expression for f is not applicable to turbulent flow.
- Using the expressions for hydraulic radius and velocity through the bed we get

$$\frac{\Delta p}{L} = \frac{3f\rho V_0^2}{\Phi_s D_p}\frac{(1-\epsilon)}{\epsilon^3} \tag{17}$$

- An empirical correlation for packed bed at high Re is the Burke-

Plummer equation

$$\frac{\Delta p}{L} = \frac{1.75\rho V_0^2}{\Phi_s D_p}\frac{(1-\epsilon)}{\epsilon^3} \tag{18}$$

4.4 Overall equation

- To cover the entire range of flow rates with one equation, it is assumed that the viscous losses and the kinetic energy losses are additive.
- This results in the overall equation for $\Delta p/L$ which is known as Ergun equation.

$$\frac{\Delta p}{L} = \frac{150\mu V_0}{(\Phi_s D_p)^2}\frac{(1-\epsilon)^2}{\epsilon^3} + \frac{1.75\rho V_0^2}{\Phi_s D_p}\frac{(1-\epsilon)}{\epsilon^3} \tag{19}$$

- Note that the first term depends on μ indicating the effect of viscosity.
- The second term contains ρ indicating its effect on kinetic losses.
- A small change in ϵ has a very large effect on Δp, which makes

it very difficult to predict Δp accurately.

4.5 Minimum fluidization velocity

At fluidization, the gravity force on the particles in the bed must be balanced by the drag, buoyancy. More specifically the pressure drop across the bed times the cross-sectional area must equal the force of gravity minus the buoyancy force.

$$\Delta P S = (1-\epsilon) L S (\rho_p - \rho) g$$
$$\Rightarrow \quad \frac{\Delta P}{L} = (1-\epsilon)(\rho_p - \rho) g \tag{20}$$

At incipient fluidization $\epsilon = \epsilon_M$. Also from Ergun equation we get

$$\frac{\Delta P}{L} = \frac{150 \mu \overline{V}_{0M}}{\Phi_s^2 D_p^2} \frac{(1-\epsilon_M)^2}{\epsilon_M^3} + \frac{1.75 \rho \overline{V}_{0M}^2}{\Phi_s D_p} \frac{(1-\epsilon_M)}{\epsilon_M^3} \tag{21}$$

This gives

$$\frac{150\mu\overline{V}_{0M}}{\Phi_s^2 D_p^2}\frac{(1-\epsilon_M)}{\epsilon_M^3}+\frac{1.75\rho\overline{V}_{0M}^2}{\Phi_s D_p}\frac{1}{\epsilon_M^3}=(\rho_p-\rho)g \tag{22}$$

For very small particles, only the laminar flow term of the Ergun equation is significant. With $Re_p < 1$, the equation becomes

$$\frac{150\mu\overline{V}_{0M}}{\Phi_s^2 D_p^2}\frac{(1-\epsilon_M)}{\epsilon_M^3}\cong(\rho_p-\rho)g \tag{23}$$

giving

$$\overline{V}_{0M}=\frac{(\rho_p-\rho)g}{150\mu}\frac{\epsilon_M^3}{(1-\epsilon_M)}\Phi_s^2 D_p^2 \tag{24}$$

This equation can be used to calculate the minimum fluidization velocity if the void fraction at incipient fluidization, ϵ_M, is known.

Type of Particles	Particle Size, D_p (mm) 0.06	0.10	0.20	0.40
	Void fraction, ε_{mf}			
Sharp sand ($\phi_S = 0.67$)	0.60	0.58	0.53	0.49
Round sand ($\phi_S = 0.86$)	0.53	0.48	0.43	(0.42)
Anthracite coal ($\phi_S = 0.63$)	0.61	0.60	0.56	0.52

Table 1: Void fraction at incipient fluidization for different particles

4.6 void fraction at incipient fluidization

- ϵ_M is generally between 0.4 and 0.45 for roughly spherical particles.
- This increase slightly with decreasing particle size.

4.7 Validity of Eq.24

- Many empirical equation suggests that $\overline{V}_{0M}$ varies with somewhat less than the 2 power of the particle size and not quite inversely with viscosity.
- This may be due to the fact that the second term of the Ergun equation is neglected and dependence of ϵ_M on particle size.
- For irregular particles, the uncertainty in ϵ_M is probably the major error in predicting $\overline{V}_{0M}$.
- The dependence on D_p holds upto particles about $300\mu m$ in size.
- In many applications, the particle size is in the range $30\mu m - 300\mu m$.

4.8 $\overline{V}_{0M}$ for large particles

Fluidization is also used for particles with $D_p > 1mm$. For large particles, the laminar term becomes negligible and we get

$$\frac{1.75\rho\overline{V}_{0M}^2}{\Phi_s D_p}\frac{1}{\epsilon_M^3} = (\rho_p - \rho)g \tag{25}$$

giving

$$\overline{V}_{0M} = \sqrt{\frac{\Phi_s D_p(\rho_p - \rho)g\epsilon_M^3}{1.75\rho}} \tag{26}$$

4.9 $\overline{V}_{0M}$ for particles with unknown ϵ_M

For many systems

$$\Phi_s \epsilon_M \cong \frac{1}{14}$$
$$\frac{1-\epsilon_M}{\Phi_s^2 \epsilon_M^3} \cong 11$$

Putting these values in Eq.7 we can get

$$Re_M = \left((33.7)^2 - 0.0408 \frac{(\rho_p - \rho) g D_p^3 \rho}{\mu^2} \right)^{\frac{1}{2}} - 33.7 \qquad (27)$$

4.10 $\overline{V}_{0M}$ and terminal velocity

For laminar flow we get the terminal velocity as

$$u_t = \frac{D_p^2(\rho_p - \rho)g}{18\mu} \tag{28}$$

From Eq.24 we get $\overline{V}_{0M}$. So we have

$$\frac{u_t}{\overline{V}_{0M}} = \frac{\frac{D_p^2(\rho_p-\rho)g}{18\mu}}{\frac{(\rho_p-\rho)g}{150\mu}\frac{\epsilon_M^3}{(1-\epsilon_M)}\Phi_s^2 D_p^2} = \frac{8.33(1-\epsilon_M)}{\Phi_s^2\epsilon_M^3} \tag{29}$$

For sphere with $\epsilon_M \approx 0.45$

$$\frac{u_t}{\overline{V}_{0M}} \approx 50$$

- a bed can be operated with velocities upto 50 times of $\overline{V}_{0M}$ without much carry out
- for nonspherical particles $\Phi_s < 1$, however value of ϵ_M is greater. For $\Phi_s = 0.8$ and $\epsilon_M = 0.5$, $\frac{u_t}{\overline{V}_{0M}} = 52$. So even for nonspherical small particles in laminar flow bed can be operated with very high velocity.

For large particles

$$u_t = 1.75 \left[\frac{gD_p(\rho_p - \rho)}{\rho}\right]^{\frac{1}{2}} \tag{30}$$

Also $\overline{V}_{0M}$ is given by Eq.26. So we get

$$\frac{u_t}{\overline{V}_{0M}} = 1.75\left[\frac{gD_p(\rho_p-\rho)}{\rho}\right]^{\frac{1}{2}}\left[\frac{\Phi_s D_p(\rho_p-\rho)g\epsilon_M^3}{1.75\rho}\right]^{\frac{1}{2}}$$
$$= \frac{2.32}{\epsilon_M^{3/2}} \tag{31}$$

For $\epsilon_M \approx 0.45$

$$\frac{u_t}{\overline{V}_{0M}} = 7.7$$

- For coarse particles $\frac{u_t}{\overline{V}_{0M}}$ is much lower than fine particles.
- Entrainment may be severe for large particles.

4.11 Bed length at minimum fluidization

At minimum fluidization, the bed volume is given as

$$Volume_M = S(1 - \epsilon_M)L_M \tag{32}$$

giving mass of the particle, M_p, as

$$M_p = S(1 - \epsilon_M)L_M\rho_p \tag{33}$$

So the length of the bed is obtained as

$$L_M = \frac{M_p}{S(1 - \epsilon_M)\rho_p} \tag{34}$$

4.12 Bed length and porosity

Volume of a fluidized bed $= LS$
Fraction of volume which is solid $= (1-\varepsilon)$
Total volume of solid if it were as one piece $= LS(1-\varepsilon)$
Since the solid volume must be consereved

$$\begin{aligned} & L_1 S(1-\varepsilon_1) = L_2 S(1-\varepsilon_2) \\ \Rightarrow \quad & \frac{L_1}{L_2} = \frac{1-\varepsilon_2}{1-\varepsilon_1} \end{aligned} \tag{35}$$

5 Fluidization regimes

The appearance of a fluidized beds are quite different depending on the fluidization velocity and the fluid used. Accordingly, fluidization is categorized in a number of regimes. Although the characterization is presented in the following figure as a function of fluidization velocity, the fluid medium has an important role in determining which regime is expected to be observed for a particular fluidization process. Followings are the main types of fluidization observed in different systems.

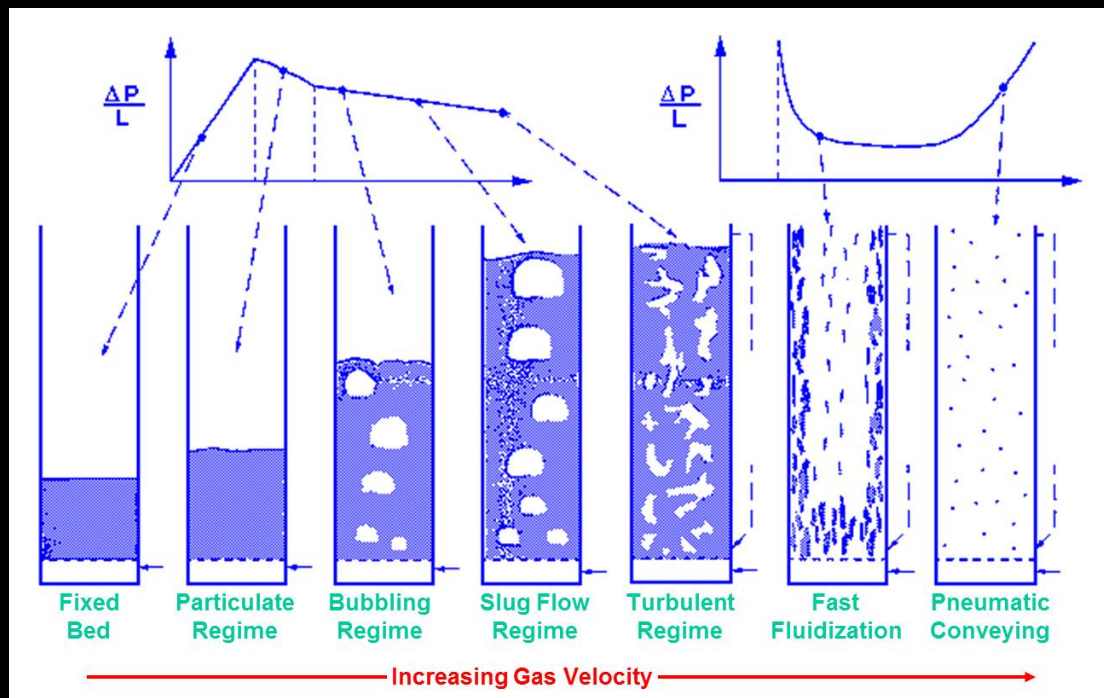


Figure 6: Fluidization regimes [Courtesy: Andre Bakker, Source:Perry's Chemical Engineers' Handbook, Adapted from Garce, Can J. Chem. Eng., 64, 353-363 (1986)].

5.1 Particulate fluidization

- Particulate fluidization is characterized by a large but uniform expansion of the bed a high velocities.
- For example, when fluidizing sand with water, the particles move farther apart and their motion becomes more rigorous as the velocity is increased, but the average bed density at a given velocity is the same in all sections of the bed.

5.2 Bubbling fluidization

- Beds of solids fluidized with air usually exhibits bubbling fluidization.
- Beyond minimum fluidization, most of the gas passes through the bed as bubbles or voids which are almost free of solids.
- Only a small fraction of the gas flows in the channels between the particles.
- The particles move erratically and are supported by the fluid, but in the space between bubbles.
- The bed is nonuniform.

5.3 Slug flow

- If a small diameter tube is used with a deep bed of solids, gas bubbles may grow until they fill the entire cross section.
- Successive bubbles then travel up the column separated by slugs of solids.

5.4 Turbulent or fast fluidization

- At very high velocities beyond the minimum fluidization, the bed expands so much that there can no longer be a dispersed bubble phase.
- The gas phase is continuous and there are small regions of high or low bed density, with rapid density fluctuations at all points in the bed.

5.5 Pneumatic conveying

- At very high velocities, all particles are rapidly entrained with the gas, but they can be separated and returned to the bed.
- There is no distinct bed of solid and the volume fraction of solid is quite low.

6 Workbook: Sizing fluidization vessel

- Problem statement:
  Catalyst pellets $5\ mm$ in diameter are to be fluidized with $45,000\ kg/h$ of air at $1\ atm$ and 80^oC in a vertical cylindrical vessel. The density of the catalyst particles is $960\ kg/m^3$; their sphericity is 0.86. If the given quantity of air is to be just sufficient to fluidize the solids, what should be the diameter of the vessel.

- Solution
  List the given information :
  - Diameter of particles,
  - Density of particles,
  - Sphericity of the particles,
  - Mass flow rate of fluid,

- Fluid is air at $1\ atm$ and 80^oC

List the unknowns to be determined :

- The diameter of the vessel

Solution procedure :

- Analysis
  - Mass flow rate of the fluid is given from which volumetric flow rate can be obtained.
  - If we know the velocity of fluid, then the cross sectional area can be obtained using the flow rate.
  - The diameter can then be estimated from the area.
- Plan
  - The condition is the minimum fluidization condition
  - The minimum fluidization velocity will be the velocity of the

fluid that can be obtained from

$$\frac{150\mu\overline{V}_{0M}}{\Phi_s^2 D_p^2}\frac{(1-\epsilon_M)}{\epsilon_M^3} + \frac{1.75\rho\overline{V}_{0M}^2}{\Phi_s D_p}\frac{1}{\epsilon_M^3} = (\rho_p - \rho)g$$

Or

$$a\overline{V}_{0M}^2 + b\overline{V}_{0M} + c = 0$$

- To use the above equation we need to know μ and ρ. For air at $1\ atm$ and 80^oC

$$\mu = 0.02 \times 10^{-3} kg/m - s$$
$$\rho = \frac{29\ kg/kmol}{22.4\ m^3/kmol}\frac{273\ ^oK}{(273+80)\ ^oK} = 1\ kg/m^3$$

- Also for this particles, ϵ_M can be assumed as 0.45.

- For the given values

$$a =$$

$$=$$

$$=$$

$$b = \frac{150\mu}{\Phi_s^2 D_p^2} \frac{(1-\epsilon_M)}{\epsilon_M^3}$$

$$=$$

$$=$$

$$
\begin{aligned}
c &= -(\rho_p - \rho)g \\
&= \\
&=
\end{aligned}
$$

So we have the equation

$$4466\overline{V}_{0M}^2 + 979.3\overline{V}_{0M} - 9408 = 0$$

Giving

$$
\begin{aligned}
\overline{V}_{0M} &= \\
&= \\
&=
\end{aligned}
$$

The volumetric flow rate of air

$$F =$$

$$= 12.5\, m^3/s$$

If the cross sectional area of the vessel is S, then the superficial velocity

$$\overline{V}_0 =$$

$$=$$

So we get

$$=$$

$$\Rightarrow \quad S =$$

$$\Rightarrow \quad S = 9.26m^2$$

If the diameter of the vessel is D, then

$$= 9.26m^2$$

$$\Rightarrow \quad D =$$

$$\Rightarrow \quad D =$$

7 Workbook: Comparison of experimental and theoretical results

- Problem statement:
  In backwashing a bed of 20-mesh to 50-mesh Dowex 50-X8 resin, the bed starts to expand when the flow rate reaches $0.4\ gal/min.ft^2$ and has expanded 45 percent at $6\ gal/min.ft^2$. Are these values consistent with fluidization theories?
- Solution
  List the given information:
  - Identify the solid particle and the fluid.
  - List the given particle properties

- ○ List the given fluid properties

- ○ List the given flow rates

- ○ List any other information provided

List the unknowns to be determined :

- ○ Are these values consistent with fluidization theories?
- ○ Which values?

- ○ At the end you have to estimate and and compare those with the given values to check whether they match or not.

Solution procedure :

- Analysis: we need to determine
  - the minimum fluidization velocity and
  - the length of the bed at a given velocity
- Plan
  - The minimum fluidization velocity can be obtained from

Or

$$a\overline{V}_{0M}^{2} + b\overline{V}_{0M} + c = 0$$

  - to determine the bed height at a given velocity we can use

$$\frac{L}{L_M} = \frac{1-\varepsilon_M}{1-\varepsilon}$$

- DO To use the above equation we need to know μ and ρ for water which are

$$\mu = 1cP = 1 \times 10^{-3} kg/m - s$$
$$\rho = 1 \times 10^{3}\ kg/m^{3}$$

Also we need the properties of the particles. For the given resin

$$D_p = 0.56mm = 0.56 \times 10^{-3} m$$
$$\rho_p = 1.24 \times 10^{3}\ kg/m^{3}$$

Also for this particles, ϵ_M can be assumed as 0.35.

For the given values

$$a =$$

$$=$$

$$=$$

$$b = \frac{150\mu}{\Phi_s^2 D_p^2} \frac{(1-\epsilon_M)}{\epsilon_M^3}$$

$$=$$

$$=$$

$$
\begin{aligned}
c &= -(\rho_p - \rho)g \\
&= \\
&=
\end{aligned}
$$

So we have the equation

$$7288\overline{V}_{0M}^{2} + 725.1\overline{V}_{0M} - 0.2352 = 0$$

Giving

$$
\begin{aligned}
\overline{V}_{0M} &= \\
&= \\
&=
\end{aligned}
$$

The flow rate of the fluid for the minimum fluidization condition is given as

$$\frac{0.4 gal}{ft^2.min} \left| \frac{\quad\quad\quad\quad}{} \right| \frac{\quad\quad}{} \left| \frac{\quad\quad}{} \right.$$

$$= 0.273 \times 10^{-3} m/s$$

We see that the given velocity at the minimum fluidization condition is very close to the theoretical value.

Next, we need to determine the expansion of the bed, for the velocity $6\ gal/ft^2.min$ which is equivalent to

$$\frac{6gal}{ft^2.min}\Bigg|\underline{\hspace{3cm}}\Bigg|\underline{\hspace{1.5cm}}\Bigg|\underline{\hspace{1.8cm}}$$

$$= 4.075 \times 10^{-3} m/s$$

We may assume a particulate fluidization here and still can use the equation for fixed bed which is

$$\frac{150\mu\overline{V}_0}{\Phi_s^2 D_p^2}\frac{(1-\epsilon)}{\epsilon^3} + \frac{1.75\rho\overline{V}_0^2}{\Phi_s D_p}\frac{1}{\epsilon^3} = (\rho_p - \rho)g$$

Note that now we know $\overline{V}_0$ and we need to determine ϵ from this equation. So we can write the equation as

$$d\frac{(1-\epsilon)}{\epsilon^3} + e\frac{1}{\epsilon^3} + f = 0$$

with

$$d = \frac{150\mu\overline{V}_0}{\Phi_s^2 D_p^2}$$

$$=$$

$$=$$

$$e =$$

$$=$$

$$=$$

$$f = -(\rho_p - \rho)g$$

$$=$$

$$=$$

So we have the equation

$$1949\frac{(1-\epsilon)}{\epsilon^3} + 51.89\frac{1}{\epsilon^3} - 2352 = 0$$
$$\Rightarrow \quad 1 - 0.974\epsilon - 1.175\epsilon^3 = 0$$

Now how to solve this equation?
If you solve this equation, yo get

$$\epsilon = 0.67$$

Then we have

$$\frac{L}{L_M} = \frac{1-\varepsilon_M}{1-\varepsilon}$$
$$= \frac{1-}{1-}$$
$$=$$

So we have a 100% expansion of the bed according to this equation. However, the given value of the bed expansion is 45%. So the fluidization equation is not consistent with the experimental value for bed expansion.

References

1. W. L. McCabe, J. C. Smith, P. Harriott. (2005). Unit Operations of Chemical Engineering, 7th Edition, McGraw Hill, New York, USA. ISBN-13: 978-0-07-284823-6
2. D. Green and R.H. Perry. 2007. Perry's Chemical Engineers' Handbook, 8th Edition, McGraw-Hill. ISBN-13: 9780071422949
3. J. F. Richardson, J. H. Harker, J. R. Backhurts (2002) Coulson and Richardson's Chemical Engineering - Particle Technology and Separation Processes, Vol. 2, Fifth Ed., Butterworth-Heinemann, Oxford, UK. ISBN 0 7506 4445 1