PROC 5071:
Process Equipment Design I

Size and Shape of Solid Particle

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1 Properties of solids particles

1.1 Why properties of solid particles are important?

- Materials are either solids or fluids.
- Process industries encounter handling of both solids and fluids.
- In process industries, the most relevant of solids are the small particles.
- An understanding of the characteristics of masses of particulate solid in designing process and equipment is necessary for dealing with streams containing such solids

1.2 Characterization of solids

- Transportation and flow properties are involved in unit operations
- In terms of flow properties, fluids are often characterized based on
  - density and
  - viscosity
- What distinguishes fluids from solids are their shapelessness
- Solids are often characterized in terms of their
  - shape and
  - size
- However, other properties are also important
  - Density - fluidization
  - Hardness, toughness, stickiness - crushing and grinding
  - Thermal conductivity - heat transfer

2 Size of particles

2.1 How to define size?

- We often define size in terms of big and small
- What is big for a processing facility is too small for a mining operation
- What is small for a process facility is too big for a nano-operation

2.2 What does size mean?

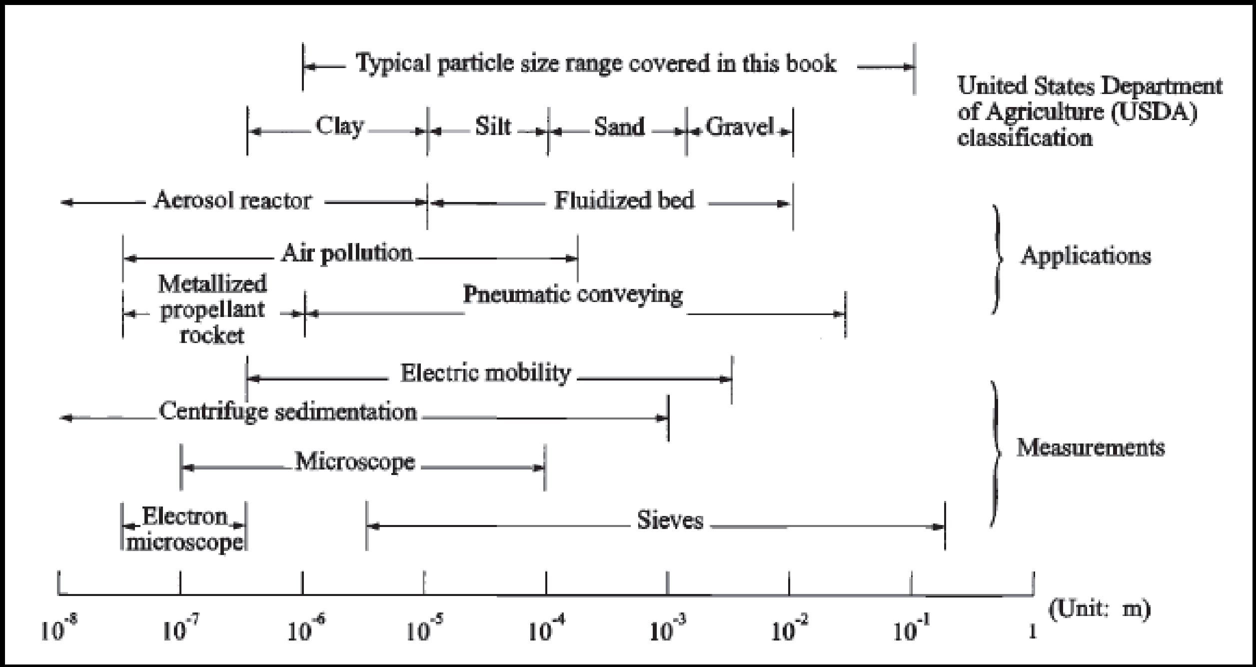

Figure 1: Magnitude of particle size in gas-solid systems [Source: Solo (1990).

- What do the above sizes mean?
- If we say diameter, then typically it refers to spherical particles
- One can also mention cubic particles with side of certain length.

2.3 Characteristic dimensions of regular shaped particles

- For regular shaped particles, say spheres or cubes, if one characteristic dimension is known, other properties say, surface area can be calculated
- For example, for a spherical particle with diameter D, the volume is $\frac{\pi}{6}D^3$ and surface area is πD^2.
- However, in real life, most particles are NOT of regular shape
- The Earth itself is not a perfect sphere; grains, fruits all are irregular in shape; and the human body itself is a vivid example of irregualr shape object.
- How to define size of irregular shape particles.

2.4 Characteristic dimensions of irregular shaped particles

- The fundamental problem is how to define a three dimensional irregular shaped particle by one dimension.
- In practice, it's not possible. However, by defining a size and a property defining its shape, a particle can be characterized.
- First the shape needs to be defined.

3 Shape of particles

3.1 Uniqueness of the sphere

- Symmetry is a unique feature of the sphere
- A sphere, despite being three dimensional, can be characterized by just one number, its diameter.
- Also, the sphere has the smallest surface area to volume ratio. Surface area is one of the important particle property for chemical and process industries.
- The sphere is taken as standard.
- Particle shapes are compared with the sphere by defining their sphericity.

3.2 Sphericity

- The sphericity is defined as

$$\Phi_s = \frac{s_{sph}}{s_p} \tag{1}$$

s_{sph}: surface area of sphere of same volume as particle
s_p: surface area of particle

- Defining v_p as volume of one particle and v_{sph} as volume of the sphere, where $v_{sph} = v_p$

$$\Phi_s = \frac{s_{sph}/v_{sph}}{s_p/v_p} \tag{2}$$

- For a sphere, $s_{sph} = \pi D_{sph}^2$ and $v_{sph} = \frac{\pi}{6} D_{sph}^3$ where D_{sph} is the diameter of the sphere.

- So we have $\frac{s_{sph}}{v_{sph}} = \frac{6}{D_{sph}}$ giving

$$\Phi_s = \frac{6/D_{sph}}{s_p/v_p} \tag{3}$$

- For irregular shaped particles, an equivalent diameter, D_p, is defined as the diameter of a sphere with the same volume as that of the particle. So we have $D_{sph} = D_p$ which gives

$$\Phi_s = \frac{6/D_p}{s_p/v_p} \tag{4}$$

- Note that sphericity of particles of a particular shape is independent of their sizes
  - sphericity of all cubes are the same

3.3 Notes on equivalent/nominal diameter

- An equivalent diameter is defined as the diameter of a sphere of equal volume.
- However, sometimes equivalent diameter of certain shapes are defined arbitrarily; e.g. equivalent diameter of a cube is defined as the length of one side.
- Sometimes the second-largest dimension is taken as equivalent diameter; e.g. equivalent diameter of cylinders are taken as their diameter.
- For fine granular materials, equivalent diameter is determined based on screen size.

3.4 Workbook: Sphericity of particles

Determine the sphericity of

1. a sphere with diameter of 2 cm.
2. a cube with one side D_p cm.
3. a cylinder with diameter of D_p cm and $L = D_p$.

3.5 Workbook: Sphericity of hemispheres

You have a sphere with diameter D. if you cut the sphere into two halves, determine the sphericity of each of the hemispheres.

3.6 What's the use of sphericity

- Question arises that if we need surface area and volume to estimate the sphericity, then what's its use?
- In reality, the sphericity is used to calculate other properties, e.g. surface area.
- Sphericity is independent of size of particles having the same shape.
- If you know or determine the sphericity of one particle, you can use it for bulk particles with different sizes but the same shape.
- Many naturally existing particles, e.g. minerals, may have different sizes but have the same shape.
- Crystals may grow to various sizes while having the same shape.

Table 1: Sphericity of miscellaneous materials (Source: Perry's Chemical Engineering Handbook).

Materials	Sphericity
Spheres, cubes, short cylinders ($L = D_p$)	1.0
Raschig rings ($L = D_p$)	
$L = D_o$, $D_i = 0.5D_o$	0.58
$L = D_o$, $D_i = 0.75D_o$	0.33
Berl saddles	0.3
Ottawa sand	0.95
Rounded sand	0.83
Coal dust	0.73
Flint sand	0.65
Crushed glass	0.65
Mica flakes	0.28

3.7 Volume shape factor

- Volume of any particle is proportional to the its diameter cubed

$$v_p \propto D_p^3 \tag{5}$$

- For a given shape we get

$$v_p = aD_p^3 \tag{6}$$

- Here a is called the volume shape factor. For a sphere $a = \frac{\pi}{6} = 0.5236$, for a cube $a = 1$.
- The volume shape factor is independent of size; it depends on the shape.

4 Properties of bulk particles

- We determine the size and shape of particles to calculate other particle characteristics, e.g. surface area, or properties of a mass of particles.
- For irregular shape particles, it is not straightforward to determine surface area of particles.
- For many particles, we can determine the size by screen analysis.
- Also for certain particles, the sphericity is known.
- Now, how to determine the total surface area of a mass of particles with known particle density?

4.1 Surface area of a bulk of equal size irregular shape particles

- Total surface area of a mass of particles

$$A = N \times s_p \tag{7}$$

where, N is the number of particles and s_p is the surface area of one particle.

- The number of particles can be obtained by dividing the total mass by the mass of one particle

$$N = \frac{m}{\rho_p v_p} \tag{8}$$

- So we have

$$A = \frac{m}{\rho_p v_p} \times s_p \tag{9}$$

- However, it is not straightforward to measure the surface area or volume of individual particles because of their irregular shape.
- If the sphericity, Φ_s and the diameter of particles, D_p are known, we can relate s_p and v_p to Φ_s

$$\begin{aligned} \Phi_s &= \frac{6/D_p}{s_p/v_p} \\ \frac{s_p}{v_p} &= \frac{6}{\Phi_s D_p} \end{aligned} \tag{10}$$

- This gives

$$\begin{aligned} A &= \frac{m}{\rho_p} \times \frac{s_p}{v_p} \\ &= \frac{6m}{\Phi_s \rho_p D_p} \end{aligned} \tag{11}$$

4.2 Properties of a bulk of particles with different sizes

- A bulk containing particles with different sizes, the size distribution is normally expressed as a histogram as shown below.
- The histogram represents mass fraction of particles of different sizes.
- An alternate representation depicting cumulative mass fraction smaller than stated sizes is also used.

4.3 Surface area of a bulk of particles with different sizes

- The total surface area of a mass of particles can be represented as a sum of the surface areas of masses of different sizes.
- For a size $\overline{D}_{p_i}$, if the mass fraction of particles is x_i, then the surface area of all particles of this size is

$$A_i = \frac{6m_i}{\Phi_s \rho_p \overline{D}_{p_i}} \tag{12}$$

- If there are n number of sizes, total area is

$$A = \sum_{i=1}^{n} \frac{6m_i}{\Phi_s \rho_p \overline{D}_{p_i}} \tag{13}$$

- The specific surface area i.e. the surface area per unit mass of particles is given by

$$A_w = \frac{\frac{6}{\Phi_s \rho_p} \sum_{i=1}^{n} \frac{m_i}{\overline{D}_{p_i}}}{\sum_{i=1}^{n} m_i} \tag{14}$$

- If the total mass is m, then $m_i = m x_i$ and we get

$$A_w = \frac{\frac{6}{\Phi_s \rho_p} \sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}}}{\sum_{i=1}^{n} x_i} \tag{15}$$

- Note that if all of the screens are considered then $\sum_{i=1}^{n} x_i = 1$ and the equation simplifies to

$$A_w = \frac{6}{\Phi_s \rho_p} \sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}} \tag{16}$$

5 Average size of bulk particles

5.1 What does "mean" mean?

- Typically when we say mean, we refer to the arithmetic mean; for example, the arithmetic mean of three numbers x,y, and z is $\frac{x+y+z}{3}$
- The arithmetic mean is relevant any time several quantities add together to produce a total. The arithmetic mean answers the question, "if all the quantities had the same value, what would that value have to be in order to achieve the same total?"
- The arithmetic mean is applicable when the quantities are added e.g. profit. You can get a mean value of the profit per year by taking arithmetic mean over several years.

- However, arithmetic mean may not be applicable for all cases. For example, if the numbers represent quantities which are multiplied as in the case of rate of growth.
- If the growth for three successive years are 0%, 10% and 50%, an arithmetic mean of 20% will not be correct estimate of the growth.
- In this case the value was multiplied by 1.0, 1.1 and 1.5 in the three years. To get the constant factor to be multiplied by each year in order to achieve the same effect as multiplying by 1.1 one year, 1.1 the next, and 1.5 the third, you need to get the mean as $(1.0 \times 1.1 \times 1.5)^{1/3}$ which is called the geometric mean.

5.2 Different ways to define average size

There are a number of ways to define the average particle size.

1. Surface mean diameter
2. Arithmetic mean diameter
3. Mass mean diameter
4. Volume mean diameter

5.3 Surface mean diameter

- defined as the diameter of the same number of equal size particles that would give the same surface area as that of the mass.
- If there were only one size particles with diameter D_s, the specific surface area would be

$$A_w = \frac{6}{\Phi_s \rho_p \overline{D}_s} \tag{17}$$

- The specific surface of the mass is given by

$$A_w = \frac{\frac{6}{\Phi_s \rho_p} \sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}}}{\sum_{i=1}^{n} x_i} \tag{18}$$

- Equating these two

$$\overline{D}_s = \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}}} \tag{19}$$

- If all of the screens are considered, then

$$\overline{D}_s = \frac{1}{\sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}}} \tag{20}$$

5.4 Arithmetic mean diameter

- defined as the weighted average of the diameters where the number of particles is used as the weighting factor.

$$\overline{D}_N = \frac{\sum_{i=1}^{n} N_i D_{p_i}}{\sum_{i=1}^{n} N_i} \tag{21}$$

5.5 Mass mean diameter

- defined as the weighted average of the diameters where the mass fraction of particles belonging to the stated size is used as the weighting factor.

$$\overline{D}_w = \frac{\sum_{i=1}^{n} x_i D_{p_i}}{\sum_{i=1}^{n} x_i} \tag{22}$$

5.6 Volume mean diameter

- defined as the diameter of the same number of equal size particles that would give the same volume as that of the mass.
- If there were only one size particles with diameter $\overline{D}_v$, the volume would be

$$v = N \times a\overline{D}_v^3 \tag{23}$$

- The volume of the mass is given by

$$v = \frac{\sum_{i=1}^{n} m_i}{\rho_p} \tag{24}$$

- Equating these two

$$N \times a\overline{D}_v^3 = \frac{\sum_{i=1}^{n} m_i}{\rho_p} \tag{25}$$

- N is related to the known parameters as

$$N = \sum_{i=1}^{n} \frac{m_i}{\rho_p a \overline{D}_{p_i}^3} \tag{26}$$

- Using these relations and $x_i = m_i/m$ we get

$$D_v = \left[\frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}^3}} \right]^{\frac{1}{3}} \tag{27}$$

6 Points to ponder

- For an irregular shape particle, what does ‘size’ mean?
- Why is ‘shape’ important for particles?
- Why do we consider ‘sphericity’ and what is its use?
- Why do we need to define mean diameter in different ways?

References

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