PROC 5071:
Process Equipment Design I

Size and Shape of Particles

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1 Properties of solids particles

1.1 Why properties of solid particles are important?

- Materials are either solids or fluids.
- Process industries encounter handling of both solids and fluids.
- In process industries, the most relevant of solids are the small particles.
- An understanding of the characteristics of masses of particulate solid in designing process and equipment is necessary for dealing with streams containing such solids

1.2 Characterization of solids

- Transportation and flow properties are involved in unit operations
- In terms of flow properties, fluids are often characterized based on
  - density and

- viscosity
- What distinguishes fluids from solids are their shapelessness
- Solids are often characterized in terms of their
  - shape and
  - size
- However, other properties are also important
  - Density - fluidization
  - Hardness, toughness, stickiness - crushing and grinding
  - Thermal conductivity - heat transfer

2 Size of particles

2.1 How to define size?

- We often define size in terms of big and small
- What is big for a processing facility is too small for a mining operation
- What is small for a process facility is too big for a nano-operation

2.2 What does size mean?

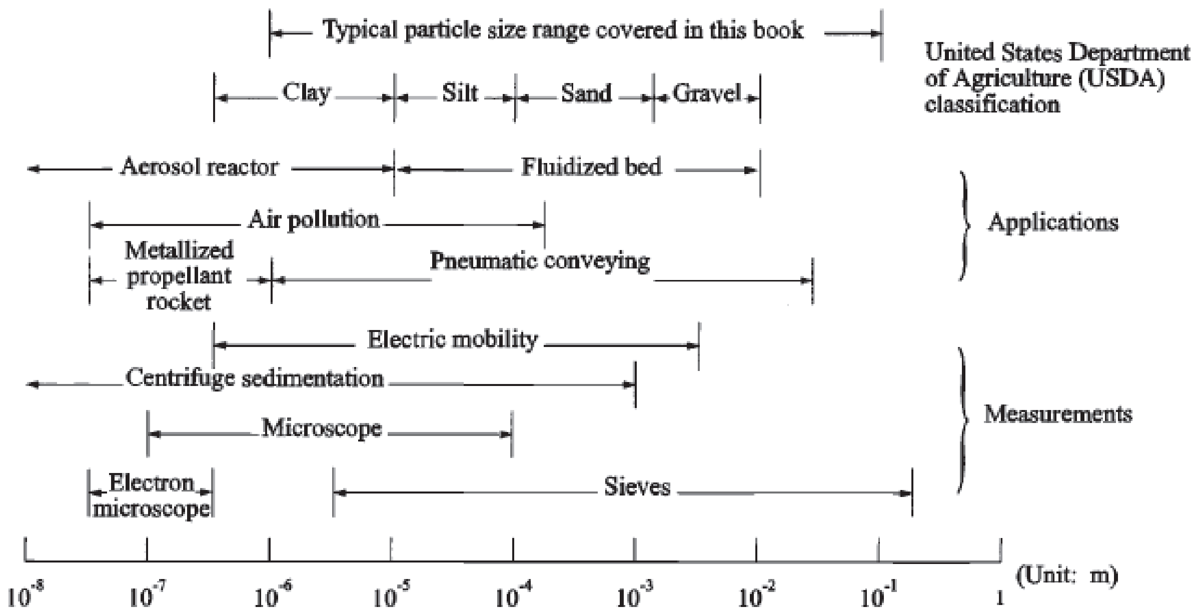

Figure 1: Magnitude of particle size in gas-solid systems [Source: Solo (1990).

- What do the above sizes mean?
- If we say diameter, then typically it refers to spherical particles
- One can also mention cubic particles with side of certain length.

2.3 Characteristic dimensions of regular shaped particles

- For regular shaped particles, say spheres or cubes, if one characteristic dimension is known, other properties say, surface area can be calculated

- For example, for a spherical particle with diameter D, the volume is $\frac{\pi}{6}D^3$ and surface area is πD^2.
- However, in real life, most particles are NOT of regular shape
- The Earth itself is not a perfect sphere; grains, fruits all are irregular in shape; and the human body itself is a vivid example of irregualr shape object.
- How to define size of irregular shape particles.

2.4 Characteristic dimensions of irregular shaped particles

- The fundamental problem is how to define a three dimensional irregular shaped particle by one dimension.
- In practice, it's not possible. However, by defining a size and a property defining its shape, a particle can be characterized.
- First the shape needs to be defined.

3 Shape of particles

3.1 Uniqueness of the sphere

- Symmetry is a unique feature of the sphere
- A sphere, despite being three dimensional, can be characterized by just one number, its diameter.
- Also, the sphere has the smallest surface area to volume ratio. Surface area is one of the important particle property for chemical and process industries.
- The sphere is taken as standard.
- Particle shapes are compared with the sphere by defining their sphericity.

3.2 Sphericity

- The sphericity is defined as

$$\Phi_s = \frac{s_{sph}}{s_p} \tag{1}$$

s_{sph}: surface area of sphere of same volume as particle
s_p: surface area of particle

- Defining v_p as volume of one particle and v_{sph} as volume of the sphere, where $v_{sph} = v_p$

$$\Phi_s = \frac{s_{sph}/v_{sph}}{s_p/v_p} \tag{2}$$

- For a sphere, $s_{sph} = \pi D_{sph}^2$ and $v_{sph} = \frac{\pi}{6} D_{sph}^3$ where D_{sph} is the diameter of the sphere.
- So we have $\frac{s_{sph}}{v_{sph}} = \frac{6}{D_{sph}}$ giving

$$\Phi_s = \frac{6/D_{sph}}{s_p/v_p} \tag{3}$$

- For irregular shaped particles, an equivalent diameter, D_p, is defined as the diameter of a sphere with the same volume as that of the particle. So we have $D_{sph} = D_p$ which gives

$$\Phi_s = \frac{6/D_p}{s_p/v_p} \tag{4}$$

- Note that sphericity of particles of a particular shape is independent of their sizes
  - sphericity of all cubes are the same

3.3 Notes on equivalent/nominal diameter

- An equivalent diameter is defined as the diameter of a sphere of equal volume.
- However, sometimes equivalent diameter of certain shapes are defined arbitrarily; e.g. equivalent diameter of a cube is defined as the length of one side.
- Sometimes the second-largest dimension is taken as equivalent diameter; e.g. equivalent diameter of cylinders are taken as their diameter.
- For fine granular materials, equivalent diameter is determined based on screen size.

3.4 Workbook: Sphericity of particles

Determine the sphericity of

1. a sphere with diameter of 2 cm.
2. a cube with one side D_p cm.
3. a cylinder with diameter of D_p cm and $L = D_p$.

3.5 Workbook: Sphericity of hemispheres

You have a sphere with diameter D. if you cut the sphere into two halves, determine the sphericity of each of the hemispheres.

3.6 What's the use of sphericity

- Question arises that if we need surface area and volume to estimate the sphericity, then what's its use?
- In reality, the sphericity is used to calculate other properties, e.g. surface area.
- Sphericity is independent of size of particles having the same shape.
- If you know or determine the sphericity of one particle, you can use it for bulk particles with different sizes but the same shape.
- Many naturally existing particles, e.g. minerals, may have different sizes but have the same shape.
- Crystals may grow to various sizes while having the same shape.

Table 1: Sphericity of miscellaneous materials (Source: Perry's Chemical Engineering Handbook).

Materials	Sphericity
Spheres, cubes, short cylinders ($L = D_p$)	1.0
Raschig rings ($L = D_p$)	
$L = D_o$, $D_i = 0.5D_o$	0.58
$L = D_o$, $D_i = 0.75D_o$	0.33
Berl saddles	0.3
Ottawa sand	0.95
Rounded sand	0.83
Coal dust	0.73
Flint sand	0.65
Crushed glass	0.65
Mica flakes	0.28

3.7 Volume shape factor

- Volume of any particle is proportional to the its diameter cubed

$$v_p \propto D_p^3 \tag{5}$$

- For a given shape we get

$$v_p = aD_p^3 \tag{6}$$

- Here a is called the volume shape factor. For a sphere $a = \frac{\pi}{6} = 0.5236$, for a cube $a = 1$.
- The volume shape factor is independent of size; it depends on the shape.

4 Properties of bulk particles

- We determine the size and shape of particles to calculate other particle characteristics, e.g. surface area, or properties of a mass of particles.
- For irregular shape particles, it is not straightforward to determine surface area of particles.
- For many particles, we can determine the size by screen analysis.
- Also for certain particles, the sphericity is known.
- Now, how to determine the total surface area of a mass of particles with known particle density?

4.1 Surface area of a bulk of equal size irregular shape particles

- Total surface area of a mass of particles

$$A = N \times s_p \tag{7}$$

where, N is the number of particles and s_p is the surface area of one particle.

- The number of particles can be obtained by dividing the total mass by the mass of one particle

$$N = \frac{m}{\rho_p v_p} \tag{8}$$

- So we have

$$A = \frac{m}{\rho_p v_p} \times s_p \tag{9}$$

- However, it is not straightforward to measure the surface area or volume of individual particles because of their irregular shape.
- If the sphericity, Φ_s and the diameter of particles, D_p are known, we can relate s_p and v_p to Φ_s

$$\begin{aligned} \Phi_s &= \frac{6/D_p}{s_p/v_p} \\ \frac{s_p}{v_p} &= \frac{6}{\Phi_s D_p} \end{aligned} \tag{10}$$

- This gives

$$A = \frac{m}{\rho_p} \times \frac{s_p}{v_p} = \frac{6m}{\Phi_s \rho_p D_p} \tag{11}$$

4.2 Properties of a bulk of particles with different sizes

- A bulk containing particles with different sizes, the size distribution is normally expressed as a histogram as shown below.
- The histogram represents mass fraction of particles of different sizes.
- An alternate representation depicting cumulative mass fraction smaller than stated sizes is also used.

4.3 Surface area of a bulk of particles with different sizes

- The total surface area of a mass of particles can be represented as a sum of the surface areas of masses of different sizes.

- For a size $\overline{D}_{p_i}$, if the mass fraction of particles is x_i, then the surface area of all particles of this size is

$$A_i = \frac{6m_i}{\Phi_s \rho_p \overline{D}_{p_i}} \tag{12}$$

- If there are n number of sizes, total area is

$$A = \sum_{i=1}^{n} \frac{6m_i}{\Phi_s \rho_p \overline{D}_{p_i}} \tag{13}$$

- The specific surface area i.e. the surface area per unit mass of particles is given by

$$A_w = \frac{\frac{6}{\Phi_s \rho_p} \sum_{i=1}^{n} \frac{m_i}{\overline{D}_{p_i}}}{\sum_{i=1}^{n} m_i} \tag{14}$$

- If the total mass is m, then $m_i = m x_i$ and we get

$$A_w = \frac{\frac{6}{\Phi_s \rho_p} \sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}}}{\sum_{i=1}^{n} x_i} \tag{15}$$

- Note that if all of the screens are considered then $\sum_{i=1}^{n} x_i = 1$ and the equation simplifies to

$$A_w = \frac{6}{\Phi_s \rho_p} \sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}} \tag{16}$$

5 Average size of bulk particles

There are a number of ways to define the average particle size.

1. Surface mean diameter
2. Arithmetic mean diameter
3. Mass mean diameter
4. Volume mean diameter

5.1 Surface mean diameter

- defined as the diameter of the same number of equal size particles that would give the same surface area as that of the mass.
- If there were only one size particles with diam-

eter D_s, the specific surface area would be

$$A_w = \frac{6}{\Phi_s \rho_p \overline{D}_s} \tag{17}$$

- The specific surface of the mass is given by

$$A_w = \frac{\frac{6}{\Phi_s \rho_p} \sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}}}{\sum_{i=1}^{n} x_i} \tag{18}$$

- Equating these two

$$\overline{D}_s = \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}}} \tag{19}$$

- If all of the screens are considered, then

$$\overline{D}_s = \frac{1}{\sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}}} \tag{20}$$

5.2 Arithmetic mean diameter

- defined as the weighted average of the diameters where the number of particles is used as the weighting factor.

$$\overline{D}_N = \frac{\sum_{i=1}^{n} N_i D_{p_i}}{\sum_{i=1}^{n} N_i} \quad (21)$$

5.3 Mass mean diameter

- defined as the weighted average of the diameters where the mass fraction of particles belonging to the stated size is used as the weighting factor.

$$\overline{D}_w = \frac{\sum_{i=1}^{n} x_i D_{p_i}}{\sum_{i=1}^{n} x_i} \quad (22)$$

5.4 Volume mean diameter

- defined as the diameter of the same number of equal size particles that would give the same volume as that of the mass.
- If there were only one size particles with diameter $\overline{D}_v$, the volume would be

$$v = N \times a\overline{D}_v^3 \quad (23)$$

- The volume of the mass is given by

$$v = \frac{\sum_{i=1}^{n} m_i}{\rho_p} \tag{24}$$

- Equating these two

$$N \times a\overline{D}_v^3 = \frac{\sum_{i=1}^{n} m_i}{\rho_p} \tag{25}$$

- N is related to the known parameters as

$$N = \sum_{i=1}^{n} \frac{m_i}{\rho_p a \overline{D}_{p_i}^3} \tag{26}$$

- Using these relations and $x_i = m_i/m$ we get

$$D_v = \left[\frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} \frac{x_i}{\overline{D}_{p_i}^3}} \right]^{\frac{1}{3}} \tag{27}$$

6 Points to ponder

- For an irregular shape particle, what does ‘size’ mean?
- Why is ‘shape’ important for particles?
- Why do we consider ‘sphericity’ and what is its use?
- Why do we need to define mean diameter in different ways?

References

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