REPORT ON THE MEASUREMENT OF REAL OUTPUT AND THE IMPLICATIONS FOR MEASURING PRODUCTIVITY GROWTH AND TECHNICAL PROGRESS

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1. Introduction[i]

Theoretical and practical problems of measuring output in the economy have produced a lengthy and detailed literature. Since we do not wish to replicate arguments that have been extensively debated in the literature, many points will be simply reviewed.[ii] In addition, a certain structure has to be imposed on the extensive material so that particular points can be clearly understood independently of the complex whole. This report is particularly concerned with the problem of defining a measure of output for individual industries in an economy. However, there appears to be considerable confusion arising from considerations that are most relevant for measuring output or income for the whole economy with those for a single industry. The first section will establish the three types of output measures that we will consider. This will be followed by a review of the measurement of output for the whole economy. The remaining sections contain a more detailed consideration of industry output. A theoretical discussion of real value-added is followed by an analysis of recent Canadian practices in real output measurement. These are done without any concern for technical change and productivity. The final two sections introduce technical change and productivity.

2. Real Output: Technology and Welfare

Throughout this report, it will be useful to assume that there is a production technology that describes how inputs can be combined to produce output or outputs. For simplicity we will represent the technology in the form

$$Q = f(K,L,M) \tag{1}$$

where, Q = gross output per period
K = capital service input per period
L = labour service input per period

M = materials or non-labour service
input per period

To avoid notational complexity, we have assumed that there are only three inputs. Nothing of substance is lost and each input can be thought of as a vector in a many input model. the concept of a production technology while basic to almost all economic analysis of production must be carefully applied. Two particular questions arise, one of which we will largely ignore and the other we will have to investigate. The notion of a production function is an abstraction from the details of reality but nothing useful can be gained from remaining at the level of minute detail of reality. Economists have applied the concept in broad and narrow cases ranging from the whole economy to specific engineering processes within a plant. In this report we will not consider the detailed question of aggregating production function for some arbitrary micro level to some higher level. Green (1964) and Fisher (1969) provide excellent discussions of this problem. For practical reasons we will consider one aspect, duplication. Appendix E contains this discussion.

There are only three concepts of output that will be considered.[iii] The three are gross output, Q, deliveries to final demand, QF, and real value-added, VQ. Deliveries to final demand are equal to gross output minus deliveries to intermediate demand, QM

(2) $$QF = Q-QM$$

Real value added is a measure of primary input use.

If we wish to study the behaviour of producers as market conditions change due to changes in private or public behaviour, then measures of gross output are required. Any other measure but gross output will be a specialized and poor substitute. The technology relates gross output to the inputs used to produce it. Hence we must have information on gross output before producer's behaviour can be studied. There are other reasons for having alternative measures of output but these cannot be used to study the response of producers to market changes except in special cases.

Deliveries to final demand QF are of interest because economic welfare has been defined in terms of output that flows to current consumption or enhances future consumption. Real value added is primarily useful as a descriptive measure of the use of primary resources or the industrial sources of real income. As we will see below, in special cases real value-added is a measure of intermediate output that can be combined with other inputs to produce gross output.

In the report, we will concentrate on the measure of output required to analyse the behaviour of the economy, gross output flows. This is done for several reasons. First, gross output measures have been ignored in national economic statistics due to the concern for welfare based measures of aggregate output. Second, understanding changes in real value added and deliveries to final demand requires information about gross output. Even if our aim is to study these alternative measures, their behaviour will have to be explained in terms of gross output. Finally, disaggregated industrial statistics and productivity studies have erroneously been based on real value-added and we want to explore what errors this involves.

An ideal system of production statistics might provide information on all three output measures. As an illustration, the input-output tables show how the three concepts are interrelated. What is objectionable about past input-output tables is the fixed coefficient assumption. If we are interested in the welfare of the nation and hence QF, we will only be able to study this variable if we can integrate it into a supply of gross output that is split between demands for deliveries to final demand and deliveries to other firms or industries. Information on deliveries to final demand is a useful descriptive statistic but we will not be able to understand why QF changes by trying to relate QF to input usage. We have to use information on a production technology that relates gross output to all inputs.

What has to be carefully considered is the questions that information on QF, QV, QM can be expected to answer. Second, we want to know how these measures of output relate to the gross output concept, not simply as a definition but rather in terms of the questions that can be asked. Of particular interest are the questions concerning the relationship of these measures to the use of inputs.

The interesting fact is that until quite recently there have been few attempts to link QF for a firm or an industry to the conditions of production or inputs in an industry. With the exception of the work on input-output tables there is little information on QF and input usage at disaggregated levels. The input-output framework relates QF to inputs only as a part of Q and there is no technical relationship between QF and input usage independent of gross output. It is only the very special assumption of fixed input-output coefficients for intermediate goods that permits a simple relationship to exist between QF and Q and primary or other inputs.

Input-output research has been extended in various ways to allow for variable coefficients but there is still a paucity of evidence on the effects of prices on the demand for intermediate inputs and the supplies of output to intermediate demand.

At the level of whole economy, production functions relating aggregate deliveries to final demand; e.g. GNP in constant dollars, to primary inputs have been studied but this has not been extended down to the industry level, NAICS major division. Some rather severe assumptions have to be made about (a) the division of gross output into QF and QM and (b) the use of intermediate materials before one can analyse the relationship of a production function of the form $QF = h(K,L)$ and the more usual $Q = f(K,L,M)$.[iv] More detailed information on QF at the industry level would be very interesting since QF is the variable that corresponds directly to the demand items included in GNE. We realize that the collection of this information is not easy, but we hope that the continuing work on input-output will provide data suitable for a more complete analysis of the links between the different output concepts and intermediate demands.

The income side of GNP is measured by summing up payments to capital and labour. This for a firm is its value added. Since sales and purchases by firms from firms must cancel, the sum of value added across all firms will equal the sum of deliveries to final demand across all firms if there are no indirect taxes. This is simply the standard method of measuring GNP. Value added does provide information on the amount of income generated in a particular firm or industry. Any question that involves this type of information in its answer will find the data useful.

In sharp contrast to QF, attempts have been made to use value added as a starting point for a consideration of a number of questions concerning output, productivity and the use of inputs in particular firms or industries. This practice has become relatively common as disaggregated measures of output by industry were developed. The fundamental question is what relationships might exist between value added and the growth and change in gross output and primary input use. We will argue that real value added is of limited use except in special cases as a measure of real output.

In an economy with interdependencies between firms and industries the task of isolating the contribution of particular industries was probably the motivation for real value-added. However, it would be erroneous to believe that any concept of real value added can in fact accurately reflect the output or contribution to output of particular industries.

When reviewing the procedures by which real output can be measured the very important linkages between outputs and inputs must be continuously considered. There are short-run descriptive uses for output measures isolated from any input data. However, any serious concern for measuring output so that business and government can understand the economy will have to include a consistent framework for measuring inputs as well as outputs. Although input measurement is outside the frame of reference of this report two points should be made. Our recommendations are made within a theoretically consistent treatment of inputs and outputs. Nothing that we suggest should be construed as requiring any reductions in the efforts to provide data on inputs and outputs on a consistent basis.

3. Real Output - GNP and Welfare

A basic distinction has to be made between measures of output that are of some direct welfare interest and those that are not. The development of national accounting was intimately related to the welfare theory of a nation's income, perhaps most elegantly analysed by Pigou. Net national product is the preferred measure of a nation's output in welfare terms. Crudely, net national product measures the flow of real output to particular categories of demand that are believed to be of some significance for welfare.

The welfare theory that underlies the choice of NNP has a long and distinguished history and we cannot enter into the details here. The key idea is that we wish to measure the level of consumption that is sustainable. In a simple world with no government or foreign trade this becomes the flow of real output to consumption and net investment during a given time period. The net investment represents a diversion of actual current consumption to increases in future potential consumption. If net investment were zero, then the corresponding consumption level could be sustained in the future. Resources that are used to maintain current productive capacity are not part of NNP. NNP is equivalent to our deliveries to final demand, QF, summed over all commodities.

The major problems in this area that have aroused continuing interest are associated with (a) the treatment of the government sector, (b) what deliveries are final (in some welfare sense) and (c) what range of activities are to be captured within the concept of economic welfare. The latter is some undefined, for the moment, portion of any broad notion of welfare.

Before entering into a discussion of these issues, one theoretical point should be clarified. The development of concepts such as NNP were spurred by the hope that it would be possible to provide summary quantitative measures of welfare. It has become quite clear that unambiguous measures of welfare cannot be developed from price and quantity data. This is independent of the complaints voiced more recently that environmental and non-market activities are ignored in NNP. Samuelson's classic paper (1950) provides the best statement of this result. One might wonder why there is still an interest in NNP and other imperfect welfare-based measures. While no conceivable changes in NNP will create a conceptually perfect welfare measure, there is still a great interest in having simplified quantitative measures of economic performance. We cannot see that this will abate although the demands for a wider range of data will grow, reducing the relative importance of aggregate NNP type measures. To put some perspective on issues that we will discuss below, not only is the point we have been making independent of the demands for any more broadly based welfare measures but the new measures of welfare will still be highly imperfect from a theoretical standpoint. A rather different and we believe erroneous distinction has been made by Jaszi (1958) in which he attempts to remain at a very practical level and deny the welfare orientation of NNP. The notion of final demand and a concern for its magnitude must have a firmer base than "the sum of purchases not charged to current expense by business" (Jazi, p. 56). Easterlin's comments on Jaszi's paper will adequately expand on the details of the disagreement that we have omitted here. Basically it is not very informative to define the desired concept in terms of the operational procedures used to approximately measure it when the latter have very little independent significance.

The notion of end product or final demand has been the subject of rather endless controversy. It would seem that everyone is willing to admit that, in practice, an arbitrary line must be drawn but the battle over where the line should be drawn continues. This argument has two aspects. Whatever end products may be, some will pass through a market transaction and others will not. The question of non-market transactions in end products will be discussed below. For market transactions, there are a whole host of boundary problems concerning what are intermediate products and a number of issues concerning the division of final output into consumption and investment. The largest areas of concern are expenditures by households on job related activities; clothes, transportation and food, and expenditures by businesses that are expensed as non-primary factor income; the whole range of non-monetary employee benefits. If we admit that an arbitrary line must be drawn then the appropriate strategy would seem to be the following. Over time, do we have any information that the particular division of goods into final and intermediate has become less relevant? That is, are the boundary cases growing in importance such that switching them from one side to the other will lead to large changes? An occasional monitoring of the size of the border line cases would be very useful. For example, it might be possible from the extensive traffic surveys in major Canadian cities to estimate what portion of expenditure on transportation is a business expense of households and how this has changed in the post-war period. Modest experiments, and we hope some of them can be truly modest, can provide much information about the current dividing line without requiring a large change in the standard practices before information on the consequences are available. Ruggles and Eisner as well as several others in the anniversary issue of the **Survey of Current Business,** The Measurement of Productivity (1972) provide examples of these boundary areas. The primary need is for some empirical evidence on the importance of these items. The other demand that has been extensively made is that the division between consumption and investment be altered. For market transactions the largest item is the shifting of consumer durables from consumption to investment. This requires the imputation of a service flow to consumer durables. Since the stock of durables is very high we would favour this shift. However, before altering the accounts, Statistics Canada would have to investigate the service lives of major durable goods. Given the standards that seem to be used for the imputation of residential rents, housing should be included in any review of the procedures to be used for imputations.

If we shift to non-market transactions, we are entering a vast area in which important work will undoubtedly be done in the next few decades but which cause considerable difficulties for the traditional set of National Accounts. For example, the capitalization of research and development, advertising, education and health expenditure all make perfectly good sense. On the other hand, we cannot see any simple means of implementation. A cautious approach would suggest that expenditure on these types of items should be separated from other items and that further small scale projects be encouraged to see if an implementation system can be improvised. Even if some of the techniques are crude, the importance of investment in human capital and knowledge would seem to indicate that attempts to measure their long-run impact are worthwhile.

Certainly the integration of market and non-market data on human social and economic behaviour will open up one of the most interesting areas that social science can study. It is of great interest to business since much of the expansion in manufacturing production continues to rely on the possibility of shifting production activities from one side of the market boundary to the other. Whether the growth of the service area indicates a reverse flow is difficult to tell but the shifting of the boundary is a fascinating area of continued importance. Everyone has undoubtedly made errors interpreting market phenomena because data is not available on the non-market aspects of behaviour which are jointly chosen with the market activity. The great stumbling block is the failure of households to keep records of their activities in the sense that businesses keep records. Market data are relatively cheap to obtain because firms engaged in market activity maintain records for their own purposes and for various public purposes. While economists may bemoan the failure of firms to record the correct data, the one major difference between economics and other social sciences is the availability of large quantities of data on market transactions from firms.

Data on non-market behaviour is going to be exceedingly expensive to obtain. Serious problems of methodology will arise in measuring non-quantitative variables. While we feel that non-market activity is very important not only for its' own sake but for an explanation of market behavior, developments in this area are likely to be slow. We would not like to see non-market activity ignored and there may be some areas in which non-market output can easily be measured. A rather casual forecast would suggest that some types of non-market activity can be collected on a continuous basis for the nation while others will have to be studied in special cross-section or panel samples. We do not know if Statistics Canada is the appropriate agency for this work but can see no obvious alternative.

The most prevalent suggestions for measuring non-market activities involve studies of household's use of time. Unfortunately, it will become very clear that information on the use of time is of limited value without the measurement of the other inputs and the output of the activity. However, studies of time spent in various activities is probably the easiest way to begin what will be a difficult but very useful attempt to study how the market boundary has shifted as business conditions and household time allocation change.

There is one possible valuation error that will arise if value imputations are to be made for non-market activity. A substantial fraction of the asset price of a commodity is a payment for the transaction services involved. That portion of the price should not be capitalized. The same phenomenon exists in cases involving financial transactions where the interest rate includes very large service charges. When an asset is purchased some part of the value is immediately consumed and similarly rental payments or interest payments involve service charges that presumably the owner of an asset can avoid. Very careful studies need to be made of imputations or else the errors will be large. Transactions costs of the type we have mentioned should not be included.[v]

The government sector has been a continual source of controversy ever since National Accounting began. We would prefer that some rough estimates be made for the service flow from the capital stock owned by the government so that whatever final product the government produces is the output from the use of capital and labour and not simply labour. It is impossible to be any more arbitrary than current practice which sets the final product of a substantial part of the country's capital stock at zero. Any small positive number would be closer to the correct figure and while we will not know what that is, we can come much closer than zero.

The subtler issue concerns the division of government output into final and intermediate product. To the extent that the output of the government sector is used as an intermediate product in the business sector there is double counting. At the present time, it is exceptionally difficult to allocate government expenditure between intermediate and final output. The nature of many of the goods as public goods and the lack of any concrete transactions makes the effective measurement of who is using what proportion of various outputs very difficult. We believe government final output to be substantially less than the current total expenditure but with the exception of a few areas such as defence and the criminal and civil court system, it is difficult to measure the expenditure that is on intermediate goods.[vi]

The other problem that plagues the government sector is the lack of any market for its outputs. Statistics Canada is currently involved in some very interesting and important attempts at measuring outputs in the non-commercial sector including government. We would argue that the measurement of service outputs can only be very roughly solved. In Appendix C, one of us has continued an exchange of views that was begun elsewhere concerning the measurement of non-commercial service output.

We would not ask for a radical change in the measurement of aggregate output at this time. Instead, what would be more useful, is an increased effort to find resources to investigate further the possibilities of more broadly based measures of national output.[vii] Estimates of the value of leisure, of the non-market use of time, imputations for government capital and further work on the estimation of final output distinguished from intermediate outputs and input measures create a menu of difficult problems which deserve some resources but do not yet warrant sharp shifts in current measurement practice.

4. Real Value Added

The practice of measuring industry output using real value-added has become very widespread in the post-war period. It is only in recent years that economists have carefully analysed the implications of this approach. The early work of Geary (1944) and Fabricant (1940) led to the adoption of double deflated value-added as appropriate. David (1962) (1963) was one of the early critics of double-deflated real value-added. More recently Denny (1972), Sims (1969), Arrow (1972), Bruno (1978), Khang (38) and Diewert (1971) (1973) (1974) have considered various aspects of the theoretical problems of measuring real value-added.

The basic problem with any measure of real value-added is that very special assumptions have been made in order to justify its measurement. If we maintain our three input production function,

$$(1) \qquad Q = f(K,L,M)$$

then separability must be assumed if we are going to define a real value-added concept. Assuming separability allows us to re-write the production function as

$$(3) \qquad Q = h(g(K,L),M).$$

We can define real value-added,

$$(4) \qquad VQ = g(K,L)$$

and re-write the production function,

$$(5) \qquad Q = h(VQ,M).$$

There is no assumption about how real value-added is to be measured in practice up to this point. Any concrete method of measuring real value-added assumes that the production technology is separable. What are the implications of this assumption?

The notion of separability was developed by Leontief (1936) and Green (1964) provides an excellent exposition of the results. For our purposes, separability implies that the marginal rate of substitution[viii] between capital and labour is independent of the quantity of materials. It also implies that the elasticity of substitution between materials and either capital or labour is constant and equal to the same value for each. Capital and labour can be combined into an intermediate product, called real value-added, and this in turn can be combined with purchased materials to produce the output. However, the quantity of real value-added, g(K,L), and not the quantities of labour and capital independently are what determine the output produced by a given quantity of materials. Material inputs are transformed to outputs by bundles of capital and labour which form real value-added. While some industrial processes may have a separable form, the assumption is very strong and has not been subject to much investigation. Until this is done the use of real value-added may be misleading as a measure of output.

There are many particular ways in which one can measure real value added. The most popular method is the double deflation procedure. Gross output and intermediate materials are deflated by their respective price indices. The difference in the separately deflated quantities of output and materials is called real value-added.

$$Q - M = g(K,L) \equiv VQ \tag{6}$$

If the production technology is additively separable

$$Q = g(K,L) + M \tag{7}$$

then the double deflation technique is the correct procedure. However, this additive separability requires that materials and real value-added are perfect substitutes in production. This is extremely unlikely since it implies that output could be produced with either materials or real value-added alone. As a slight generalization, we could consider a technology

$$Q = g(K,L) + aM \tag{7'}$$

The appropriate measure of real value-added is now

$$(p_Q.Q - p_M.M)/p_Q \equiv VQ \tag{8}$$

Nominal Value-added is deflated by the price of output so that the value of materials is deflated by the output price. If all inputs are used, the in equilibrium

(9) $$Q = g(K,L) - (p_M/p_Q)M$$

or (9') $$(p_Q Q - p_M M)/p_Q = g(K,L) \ .$$

These two cases of additive separability are strong examples of the separability assumption and do not differ from each other. In both cases additive separability requires that the ratio of the prices of value-added to materials be a constant. In the first case, the constant price ratio is one and in the second "a". This type of separability is closely related to Hicks' aggregation. Notice that additive separability will imply that the ratio of output to materials price is a constant provided some materials are used. It has been argued by Khang (1971), and Bruno (1978) that provided the price ratio of output to materials is constant then using Hicks aggregation theorem real value added can be defined as deflated nominal value added. A price index, $p = A.p_Q + B.p_M$ is formed where A and B are fixed weights and is used to deflate nominal value-added.

This similarity has some implications for empirical studies. It is possible to confuse the source of a constant output-materials price ratio. The ratio may be constant because of additive separability or it may be constant simply because general supply and demand conditions on all markets do not lead to any variation in the ratio. If the latter is true we are unable to obtain information about the role of materials in production and we do not know if the technology is additively separable. The latter requires another property. The ratio of the price of real value-added to materials must be constant which implies that the ratio of the output price to the price of real value-added is constant. If only p_Q/p_{VA} then real value-added may still be measured by deflation but this does not imply additive separability. The proper deflation procedure depends on the output-materials price ratio and is not necessarily the double deflation procedure.

Sims (1969) has proposed a slightly different defence of double deflated real value-added. He rightly argues that separability is at the core of the weakness of real value-added. However, he suggests that once the separability assumption is made then use of the double-deflation technique can be justified under a moderately restrictive set of conditions. He shows that if the separable technology,

$$Q = h(g(K,L),M) \tag{3}$$

is linear homogeneous then the double-deflation technique based on fixed weight price indexes is an approximation to a variable weight logarithmic index, the Divisia Index. The fixed weight base can be changed and in the limit as changes are chosen more frequently the limit is a currently weighted Divisia Index. While this is true, it does not contradict the early explicit additive separability required by double-deflation. In his final sentence, Sims states. "The question is not really 'Is double-deflation the right way to deflate value-added?' but rather, 'Does the notion of real value-added make any sense?`."

We have seen that real value-added requires a separable technology and discussed the additively separable case which is the required case for double-deflation. This case is also closely related to Hicks' aggregation which can also justify real value-added although not necessarily double-deflation.

There is one other extreme case in which rather simple measures of real value-added are possible. The additively separable case has isoquants (showing the quantities of real value-added and materials that can be used to produce a fixed output level) that are straight lines. Line (1) in Fig. 1. The other extreme case implies not that real value-added and materials are perfect substitutes but that they are perfect complements. The Leontief fixed input coefficient (case (2) in Fig. 1) is also a case in which real value-added is easily measured. Since there are no substitution possibilities in the Leontief case, the technology can be written.

(10) $$Q = \min[g)K,L),aM] .$$

If there are no inefficiencies in the use of inputs, $Q=aM = g(K,L) = VQ$ will be a measure of real value-added. Nominal value-added

(11) $$V = p_Q.M = (p_Q - p_M/a)Q = p_Q(1-aM)$$

cannot be deflated by the price of output or by the double-deflation method. Some knowledge of the input output coefficient 'a' is required if real value-added $g(K,L) = Q - aM$.

Perhaps the only serious attempt to look at these extreme models and the role of value-added in production analysis is contained in Griliches and Ringstad (1971). His detailed analysis of the role of materials in production used the 1963 Norwegian census data. He considered the two models discussed above. The Norwegian evidence provided very little support for the Leontief technology of fixed coefficients between value-added and materials. Instead the elasticity between materials and real value-added was found to be quite high. While there are definite limitations in the cross-section data that Griliches used, his results are of interest since they are relatively unique. He assumed that the real value-added technology was Cobb-Douglas

(12) $$Q = h(AK^{\alpha}L^{\beta},M)$$

which reduces to

(a) <u>perfect substitution</u> $VQ = AK^{\alpha}L^{\beta}$

(b) <u>perfect complements</u> $Q = AK^{\alpha}L^{\beta}$

He ran the following regression

(13) $$\ln Q/L = C + h \ln L + \alpha \ln(K/L) - \mu \ln(VQ/Q)$$

If model (a) is correct, the parameter, μ should equal one and if (b) is correct μ should equal zero. This is conditional on the acceptance of the Cobb-Douglas form for g(K,L) and the assumption of separability. His results were mixed. In twenty-eight industries, using the one per cent level of significance, twelve have estimates of μ that are not significantly different from one, seven are not significantly different from zero and nine are significantly different from both zero and one. It is not possible to conclude that either model is generally correct. In fact, Griliches' results could be used to support the hypotheses that the three factor Cobb-Douglas model is an improvement over either of the special additive separable cases. The three input Cobb-Douglas constrains the elasticity of substitution between value-added and materials to be one,

$$Q = AK^{\alpha}L^{\beta}M^{\tau} \tag{14}$$

In this case real value-added is still well defined since the Cobb-Douglas is a separable form. Real value-added is

$$VQ = (Q/M^{\tau}) \tag{15}$$

and its measurement requires some knowledge of τ , the output elasticity of materials. This is also the share of materials in total cost and an estimate of real value-added could be made using information on the share.

The proceeding analysis dealt only with separable cases and what is missing is any detailed studies of the acceptability of separability itself. I will report briefly on some of my own unpublished results that suggest that separability is not likely to be accepted. If these are extended in further work, then real value-added measures of output cannot be free from errors.

The study made by the authors analysed the production technology for total manufacturing in Canada from 1949-70. The inputs considered were equipment, buildings, production workers, non-production workers and materials. A non-homothetic Generalized Leontief cost function

(16) $$C = \Sigma\, \Sigma\, \alpha_{ij}(p_i p_j)^{½}.Q + \Sigma\, \alpha_{io} p_i$$

was used. The estimated equations were the factor demand equations

(17) $$X_i = \alpha_{io} + \Sigma\, \alpha_{ij}(p_j/p_i)^{½}. \qquad i=1, \ldots 5$$

Without going into the details, this set of factor demand equations does not assume any kind of separability. The separability of materials from labour and capital was rejected. Rather similar results have been found recently by Berndt and Wood (1974). For U.S. Total Manufacturing they reject separability necessary for real value-added. These are the only two direct tests of the conditions necessary for weak separability. What is needed is more tests at a lower level of aggregation.

While a consensus is developing amongst economists concerning the inadequacies of real value-added as an output measure there is an odd lack of any adequate defence of the concept. Double deflated real value-added has been widely adopted in practice. However, after rather extensive searches of the literature, no serious discussion of the theoretical limitations of the concept could be found. Geary who presents the first extensive analysis of the concept argued, "It is suggested that both series of index numbers be computed for each industry, the gross (output) index to show the trend in the amount of work done in the industry (23, p. 255). Much of what we have stated is identical except for the 'modern dress'. One might conclude that the gap between academics and statistical agencies has allowed this development to proceed.

5. Canadian Contributions to the Measurement of Industry Output

Statistics Canada has been one of the pioneers in the measurement of disaggregated industry output. The Index of Industrial Production has a very long history and the coverage of the economy has been complete since the development of the indexes of Real Domestic Product (RDP) on an annual basis from 1935 and a quarterly basis from 1946.[ix] The basic official publication, Indexes of Real Domestic Product by Industry of Origin, 1935-61 contains a detailed methodology. This can be supplemented by important unofficial papers by Berlinguette and Leacy (1961) and Garston and Worton (1968). The official position has been that real industry output will be measured by the double deflation method associated with Geary (1944) and Fabricant (1940). We will omit any detailed discussion of real value-added as a concept since this is covered in the other sections. There are a series of practical problems associated with the implementation of any output measure that need discussion. These are not simply of practical concern but have theoretical implications.

In Canada, real industry output measures were initially developed as a consistency check for the real gross national expenditure estimates constructed by deflating final expenditure categories (Statistics Canada, 61-505 p. 12, p. 205). For this purpose, the fact that the value of deliveries to final demand, i.e. gross national expenditure equals value-added summed over the economy explains the selection of real value added as the real output measure. There is no reason why deflated deliveries to final demand should equal deflated real value added in general. Accounting identities in value terms seldom imply any equality in real terms. In general one might expect that real value added grew more rapidly than real expenditure due to a rise in productivity.

The practical problems that arise in implementing output measures concern the precise definitions of value and quantity and the boundary problems associated with isolating a particular industry. Real Domestic Product is based on gross domestic product at factor cost. For most types of production analysis, the gross domestic concept is preferable. The net production concept is suitable for welfare analysis but it is useful to have a gross measure of output or resource use which includes the use of the services of the capital stock. Similarly, the national concept arbitrarily ignores certain factor input flows based on their ownership. This is not useful for positive economics although net national product is well-established as a normative measure. In general, it would seem to be most useful to have the capability of producing the gross domestic measure and the details needed to derive the alternative concepts where necessary. For example, the national concept eliminates information about the foreign resources used in the Canadian economy and includes Canadian resources used elsewhere. If we want to understand the use of resources in production, all resources have to be included. Understanding the role of foreign resources may be important but this requires a study of the supply and demand for factors from all countries on a domestic output basis. An understanding of why net national product changes will be based on information about gross national product. The narrower concept may be an appropriate summary statistic but it is seldom directly useful for positive or behavioural studies.

Many of the same comments apply to the choice of factor cost as the valuation standard. In much of the standard national accounting literature the choice of this concept is not very adequately defended. Certainly none of the cited Canadian writings provide a very detailed justification. George Jaszi perhaps has stated the rationale concisely, "Underlying the definition of national income as the sum of factor costs is a conception of it as a tool for answering questions relating to the allocation of productive resources among various uses (1958, p. 49). Factor cost is seen as a value measure related to resource use and this corresponds to our conception of either current or real value added at factor cost as measures of resource use rather than output. There are difficulties even here in defining what to include in factor costs, e.g., government interest, but the basic conventions are well established in national income accounting. The exclusion of indirect taxes from factor costs follows from the lack of any productive factor service being exchanged for this tax payment. This criterion would argue for the inclusion of indirect taxes on inputs when there are actual costs of carrying on production. The Canadian measures of value-added do include these taxes and this should be continued. Whether subsidies should be included has created some controversies. There has been some argument that subsidies should be included since they are necessary to bring forth the factor services but this would seem to be erroneous. Both subsidies and indirect taxes influence then both should be. Without delving into the intricacies of tax incidence, one comment in this area should be made. It would be possible with a small scale general equilibrium model such as Jones (1965), to delve into the effects of indirect taxes on the allocation of primary resources between industries and the resulting changes in outputs, factor prices and incomes and market output prices. The factor costs that we measure excluding indirect costs are going to be severely affected by the existence of indirect taxes and subsidies. There is no way to isolate the factor costs of primary inputs independently of the indirect tax system that is part of the economic structure that determines their quantities and prices. For the study of the behaviour of either output or primary resource use in the tobacco or alcoholic beverage industry, the indirect tax system should be a central part of the structure.

From a social welfare perspective, one can argue that consumers pay the market price including the indirect taxes. The social benefit from increased use of primary resources in an industry is the marginal product valued at the market price not at factor cost. The wedge or gap between this social and private return is created by the indirect tax system. For analysing the social benefits and costs of resource reallocation, value added at market price is needed.

Whether valuation is done at factor cost or market value is not of great importance for measuring real output. The price indexes used will vary in the two cases but the quantity of output should be the same. It is unfortunate that it is difficult to obtain accurate information on both producer's prices without any indirect taxes on outputs and the market prices paid by consumers. If we are to properly understand the effects of the indirect taxes in Canada, the two sets of prices will be required. At the moment it seems impossible to obtain market prices from producers for the products that are now covered by the industry selling price. For this reason, Statistics Canada collects prices at what is now the establishment boundary before any indirect taxes are due. The convenience of doing this suggests strongly that gross output be valued at producers' prices exclusive of the taxes that intervene between producer and purchaser. Most, but not all, purchases of intermediate inputs are exempt from the majority of the indirect taxes in Canada. Were this not true then price indexes including indirect taxes would be necessary for the deflation of the value of materials in the double deflation method of calculating real output. An ideal set of statistics would provide the user with gross output and value-added valued at market price and at producers' prices (factor cost). Until this is feasible either one will be useful for some questions and not others. The major point is that value added in current or constant dollars is a measure of resource use and not of output.

The confusion on this point can be seen by considering what Berlinquette and Leacy (1961) say about value added. Value added at factor cost "represents the industry's relative contribution to total output and is a measure of resource allocation particularly relevant to productivity studies". We would contend that the latter statement is false and the former at least misleading. Value-added is a measure of resource use and consequently not particularly suitable for output or productivity purposes.

The Berlinquette and Leacy paper provides some major examples of the differences that can arise in disaggregated indexes of gross and net output (1961, p. 255). At the aggregate level of major groups, these differences may well be eliminated. There seems to be very little difference in gross output and net output for total manufacturing from 1949-1970 and Berlinquette and Leacy find little difference in their aggregate series (1961, p. 227). The one exception may be agriculture in which in both Canada and the U.S. the rapid growth of material inputs relative to gross output and the effects of weather create a marge dispersion. This practical result has led Berlinquette and Leacy to conclude that gross outputs can be used as an indicator for net output at high levels of aggregation. We might reverse the conclusion and argue that we may feel some confidence in using real value added as a measure of gross output to the extent that the series do not move very differently. Since the double deflation method of calculating real output uses extensive resources, it would seem sensible to experiment more extensively with the use of gross output as an indicator between benchmarks to discover if any significant difference occurs at various levels of aggregation.

The paper by Garston and Worton (1968) supports the concept of factor cost although they appear to favour net domestic product at factor cost rather than gross. We have already indicated that the gross concept seems more appropriate.

While the choice of factor cost or market value will not cause any difference in a properly measured single real output, it will lead to differences in aggregation if the real outputs are not aggregated at producers prices excluding indirect taxes on outputs. This valuation is equivalent to the U.N.'s approximate basic values (1968, p. 52, equation 4.17, p. 67).

The remaining problems of implementation are predominantly difficulties associated with the lack of adequate data.

There are four problem areas which we wish to consider. They are: (a) the company establishment distinction which has plagued economic statistics, (b) the allocation of rents to the owning industry, (c) imputations and (d) the measurement of service outputs. We have treated in the section on aggregate output the broader problems associated with the imputations and measurement of government output and various non-market imputations not specifically considered here.

The establishment surveys cannot adequately measure the components of payments to capital. The residual left after deducting labour compensation and purchases of materials, fuel and electricity contains not only the service inputs not valued in the survey but also contains the errors from erroneous valuations of intra-company shipments. This has resulted in the use of company data for profits, interest, rents and capital consumption allowances. The allocation of these to establishments has been based on rather limited information. It would be useful if Statistics Canada could publish more detail on the current practices used in this area. This is simply another aspect of the regrettable failure of Statistics Canada to maintain an adequate level of documentation. While individuals in S.C. are extremely helpful one often does not know what question to ask without first having a fairly detailed description of what is done. Since it is unlikely that rather arbitrary assumptions can be avoided, these should be explicitly outlined. In particular, while profits may move with sales or shipments, the other contractual obligations, interest and rent, are not subject to the same cyclical movements but may have large discontinuities. Finally capital consumption allowances are unfortunately tied to both the tax system and the irregular movements of investment at the establishment level.[x]

The study of the behaviour of producers requires information on the use of resources owned or not. With the growth of leasing and renting in general and its wide prevalence in the service sector, a serious problem arises due to the national accounts convention regarding rents. At present all equipment and building space that is rented is recorded as income in the owning industry. The same problem extends to the capital expenditure survey which shows the expenditure in the owning industry. There is nothing inconsistent in this procedure for the purposes of showing the distribution of income by type of income payment. However, if real value added is a measure of real primary resources and in general if primary resource use is to be studied then the current practice is not very helpful. The commercial sector contains some examples of the difficulties. Ignoring the imputed residential rents, the real estate industry accumulates the rents for building space used in retail trade and offices. It is impossible to know how the capital stock is used. The current practice bears a strong resemblance to the situation in housing if there was no imputed rent calculated. The value of primary resources used in an industry varies with the contractual arrangement. It is not very helpful to suggest that one study the use of owned capital equipment and structures or to claim that the same situation exists for some labour categories, e.g. consultants. There are always arbitrary lines to be drawn in practice in implementing any conceptual scheme. The arguments concern where the line should be placed. For example it would be interesting to know the effects of extensive leasing of business machinery in areas such as computers, communications and copying. Manufacturing firms in these industries, depending on the company's accounting procedures, will show substantial rental revenue and a capital stock which perhaps in conjunction with some service personnel produces the services of various machines. Not only does this introduce an undesirable heterogeneity in the output of some manufacturing industries but it distorts the information on primary factor use in the renting industry.

There may be compelling reasons to maintain the current convention for the industrial distribution of national income based on ownership. However the use of value added as an output or a real primary resource measure is severely distorted in industries where renting is important. Moreover, even if gross output measures are available, the study of the technology in these industries is handicapped by the lack of data on the use of rented inputs. It would be very helpful if information could be provided on the rental payments made by industries. An even more heroic task would be to allocate the capital stock to using rather than owning industry. Perhaps the first task is to attempt a study of the availability of data on rents by using industry and an analysis of the relative importance of owned versus rented capital services from buildings and equipment in a few industries in the commercial service sector.

Although perhaps peripheral to the main scope of this enquiry two further comments on rents should be made. First, we would like to see more information on the detailed procedures used to estimate the imputed rents on residential housing. Very few resources seem to be devoted to this rather large imputations and it is our impression that the imputed value is too low. Second, the imputed residential rent is an example of the problems of differentiating outputs and inputs in service industries. While owner occupied housing is valued only by its capital service input, the imputed residential rent, the government sectors output is currently measured predominantly by its labour input. The growth of the governments owned capital stock and use of rented capital stock requires more explicit recognition in any output measurement. If input measures are to be used, then some imputation for the use of capital services by the government is required. Rental payments by the government should be included in the distribution of rents paid. The Canadian National Accounts provided an imputed rent on government owned buildings until the recent major revision. This has been replaced by an estimate of depreciation on government owned assets. We believe that this revision including the exclusion of capital formation in the defence sector is a mistake. Although rather arbitrary estimates will have to be made, it would be preferable to measure government output by the sum of government labour costs and an imputed value of the service flow from the government-owned capital stock. Until output measures can be developed that are independent of input measures, at least the service flow from the large government capital stock should be included. Preferably, it should not be isolated in the real estate industry but included in public administration and defence and the other non-commercial industries where government activity is important.

The service industries have been re-discovered in recent years because it is in this area, that the growth of employment and expenditure has been growing most rapidly. The problems of measuring output in these industries are very severe. One of the authors has attempted to formulate part of the required conceptual framework in Appendix C. However, the difficulties are likely to remain. Statistics Canada has embarked on an ambitious attempt to find new ways to measure output in this area and we will not provide an extended treatment here.

6. Productivity and Technical Change

The attempt to measure changes in output per unit of input (or inputs) received a definite impetus from the Soviet challenge and debates about the slow rates of growth in the 1950s. The return to relative full-employment meant that growth had to be sought in the more efficient use of resources. The famous studies by Denison, (14), (15), and Solow, (54) the extensive NBER work by Kendrick (33) and others and the classic studies by Salter (5) and Brown (8) all arose from this renewed interest. Recent work by economists has been concentrated on measures of total factor productivity and technical change. Government statistical agencies have tended to concentrate on productivity measures that are specific to one factor, e.g. labour productivity. As Stigler (67, p. 47) noted many years ago, productivity measured as the average product of an input or subset of inputs is not of major theoretical or empirical interest. The reasons for this split between the statistical agencies and professional economists are (a) the interest of some other users in measures such as labour productivity and (b) the reluctance of economists to utilize the erroneous or misleading conceptual framework of the average productivity of labour. The major issues to be discussed, aside from the difference noted above, are the problems of using value-added in studying productivity and the limited possibilities of any easy or feasible solution to the task of measuring technical change or productivity. Nadiri's excellent paper (1970) covers the current capabilities of economists to empirically study these matters.

If we maintain, for simplicity, our three inputs, capital (K), labour (L) and materials (M), any measure of aggregate inputs whether of aggregate labour or an aggregate of all inputs will be represented as a function of the disaggregated inputs. All productivity measurements involve the use of implicit or explicit production functions and we will continue to use it for analytic convenience. By a change in productivity or technical change we mean an increase in the output that can be produced by a given quantity of resources. In terms of the production function, we can write

$$Q = F(K,L,M,t) \tag{18}$$

where t is time. Now there are rather severe limitations on our capabilities of capturing the shifts in productivity over time. For the moment, we are concerned with concepts of total factor productivity, and measures of single factor productivity will be discussed later. Measures of total factor productivity using indexes involve a particular transformation of the problem of technical change. Instead of the general form given in equation (18), the usual assumption is that

$$Q = A(t).\ G(K,L,M) \tag{19}$$

The function G(K,L,M) is a particular aggregation formula in any given case. This is a restriction on the generality of equation (18) and will have quite serious consequences. The simplification assumes that the marginal rate of substitution and the elasticity of substitution between factors is unaffected by technical change. Isoquants shift in as time passes without any change in their shape or the "spacing" between output levels. The latter means that economies or diseconomies of scale are unaffected by shifts in the technology. Finally, in almost all cases the particular aggregation formula chosen has been linear homogeneous and the technology exhibits constant returns to scale. If the nature of the technological change does not satisfy these restrictions then the usual measures of total factor productivity will include errors. Total factor productivity may change because the elasticity of substitution changes or the 'spacing' of the isoquants changes. The latter will erroneously be identified as neutral shifts in the technology which they are not.

These problems were realized by Denison and he tried to allocate his initial measure of the residual total factor productivity to various sources. However there is little theoretical underpinning for his efforts. Perhaps more interesting, is the attempt by Denison to adjust the inputs for changes in efficiency. The early studies of Solow (1957) and Abromowitz (1950) had indicated that a very large proportion of the growth of output in the U.S. economy could not be accounted for by the growth of inputs. Denison showed that this startling result was due to an inadequate measurement of the input variable. This has led to extensive attempts to quantify changes in the inputs in a more complete and rigorous fashion. The Jorgenson and Griliches (1966) paper tried to show that if inputs were measured correctly then there has been almost no change in total factor productivity. This result was severely criticized and led to a lengthy exchange with Denison (1962) after Christensen and Jorgenson (1970) had revised the initial results to allow a larger rate of increase in total factor productivity. A recent contribution by Star and Hall (1973) has carried the process of a detailed investigation of quality change even further.

There has not been much agreement about the accuracy of any of these studies but some conclusions can be drawn. Any measurement of total factor productivity is by definition extremely sensitive to the measurement of the inputs. The extensive increase in the detailed specification of the inputs, see Star and Hall (1973), shifts changes in output growth from increases in productivity to changes in the quality of the inputs. The more aggregate input measures did not properly account for the increased quality of the labour used in a particular industry. It would appear that a large proportion of what early productivity studies have called increases in total factor productivity have in fact, been increases in the quality of various inputs. This leaves unexplained the reason for the increased quality of the inputs. These measures may be severely biased by the inadequacy of the input quality adjustments and the failure to find similar adjustments with which to measure quality changes in outputs. The level of sophistication towards which these studies are headed, strains the rather more limited quality of the available data.

If total factor productivity measured as the difference between indexes of outputs and inputs assumes that technical change is neutral and independent of the individual factors, what alternatives exist? The most widely used assumption in attempting to measure technical change is to assume that technical change is factor augmenting.[xi] This allows one to write the production function,

$$Q = f(a(t)K, b(t).L, c(t)M) \tag{20}$$

where a(t), b(t) and c(t) are functions of time. The labour input measured in efficiency units would be b(t).L where b(t) converts the labour input into efficiency units. The main rationale for this formulation is the possibility that information about the factor augmenting functions a(t), b(t) and c(t) can be obtained. The factor augmenting specification of technical change is still rather limiting. However, it is a considerable step forward from the productivity measures. If a(t) = b(t) = c(t) then the factor augmenting case becomes

$$Q = A(t)\, g(K,L,M)$$

which is what is assumed in the total factor productivity measures. All productivity measures make this same assumption although they may use only one input, say labour, in the denominator of the productivity formula.

If real value-added is used as a measure of output in either productivity or technical change studies what special problems does this create? Sims (1969) and Arrow (1972) have shown that the assumption of weak separability, necessary for the existence of real value-added requires that all technical change be real-value augmenting. That is, if real value-added is measured using the double deflation method then

$$(21) \qquad Q = h(A(t)g(K,L),M).$$

All increases in output that are due to technical change are imputed to real value-added. There is no evidence that this is likely. Any increase in output due to an increase in the efficiency with which materials are converted into output are attributed to real-value added. The use of real value added is a very special case of the factor augmenting assumption in which capital and labour augmentation are equal and materials augmentation is assumed to equal zero.

We have seen in this section that real value-added is of limited use in measuring productivity changes accurately unless the technology has very special features. Two points need to be emphasized. First, the measurement of productivity has proved to be extremely difficult. Second, no simple statistical measure is going to provide an adequate representation of technical change. Only expensive improvement in the available data in conjunction with complex econometric models will provide clear improvements. This will require information on gross output.

As an example of the difference that gross output and real-value added make in the measurement of total factor productivity, I will report briefly on an unpublished study by the authors.[xii]

For Canadian manufacturing it is possible to put together (with the much needed assistance of Statistics Canada) a series on gross output, real value added, capital, labour and materials in constant 1961 dollars for 1949-1970. There are, of course, a number of data problems, associated with the 1961 SIC as much as anything.

TABLE I: INDEXES OF OUTPUT AND INPUTS IN SELECTED YEARS, 1949-70
(1961 = 1.00)

	Gross Output	RVA	Primary Inputs	Total Inputs
1949	0.632	0.595	0.892	0.713
1954	0.775	0.749	0.949	0.832
1959	0.953	0.945	1.02	0.975
1964	1.275	1.279	1.119	1.228
1970	1.715	1.73	1.268	1.572

Gross output is measured as shipments plus the changes in finished goods and work in progress inventories. Real value added is measured by the double deflation technique. The two input measures are Divisia aggregates of primary and total inputs. Gross output has grown more slowly than real value added although the difference is very small. However, total inputs have grown much more quickly than real primary inputs. Both of the input measures have grown more slowly that the output measures indicating that there is some residual left as productivity. We can construct a Divisia index of productivity in Canadian manufacturing using either the gross output and total input series or the real value added and real primary input series.

TABLE II: PRODUCTIVITY IN CANADIAN MANUFACTURING

(1961 = 1.00)

	Gross Output	RVA
1949	0.886	0.666
1954	0.931	0.789
1959	0.977	0.927
1964	1.038	1.143
1970	1.091	1.365

The use of gross output permits the productivity measure to capture the rapid growth in material inputs and does not allocate any technical change in their use to the primary inputs. As we showed above, real value added productivity measures force all technical progress to be real primary input augmenting. Unless the technology is weakly separable real value-added does not measure output. If it is used to analyse productivity, we may mistakenly allocate the changes in the technology due to materials to the primary factors.

An interesting experimental calculation comparing productivity calculations when gross output and real value added are used is contained in the new U.N. National Accounts (1968, p. 66-70). The technology that they use is very specialized but it allows an exact expression to be shown for the difference with this technology.

<u>The Measurement of Productivity in Canada</u>

Canada has produced measures of productivity for the post-war period for the commercial industries (14-501, 14-201) (and some sub-aggregates) and for a number of specific industries (14-502-506). These have been admirably described, particularly the initial publication, Indexes of Output per Person Employed and Per Man hour in Canada, Commercial Non-Agricultural Industries, 1947-63, (14-501). The concept used has been real domestic product per unit of labour, either workers or man hours. Statistics Canada wrote up these reports very carefully to indicate the limitations not only of the data but of the particular productivity concept chosen. While most economists do not find labour productivity particularly interesting, it does have some general appeal as a summary descriptive measure of the changes in the capability of the population to produce output due to changes in production conditions including the availability of other factors.

The particular problems of measuring productivity using real value added have been indicated in the earlier section on productivity. The productivity work by Statistics Canada has had one extremely valuable by-product. Since careful measurement is at the core of any productivity study, the Productivity Research and Analysis section provided perhaps the most comprehensive attempt at reconciling the differences in the available series on the use of labour. These attempts by Statistics Canada to derive a consistent series of man hours and employment are very valuable. It would be useful if this work was continued and perhaps a more complete set of estimates published. Certainly we believe that a continued effort to provide an expanded and more precise analysis of the reasons for the differences in the alternative sources of labour data is exceedingly useful. Users are not capable of discovering all of the problems involved. To the extent that Statistics Canada can provide a series that is widely useful, such as the labour series developed for the productivity analysis, or better descriptions of the problems in the currently published series, this is very helpful. We would also suggest that the labour series developed for the productivity work have not had wide enough distribution although the diligent user of Statistics Canada services has undoubtedly found them.

If Statistics Canada, is going to proceed with further productivity work on its own and not through assistance provided to either private individuals, other departments or the Economic Council of Canada, then there would seem to be several important aspects that they might consider. Statistics Canada has the unique advantage of being able to know, in a way no outsider can, the weaknesses and strengths of the various collection processes. This suggests to us, that Statistics Canada. is the only group that can investigate certain data areas either by themselves or in conjunction with others. Even further disaggregation of the labour variables would be very desirable. Star and Hall (1973) has recently published a paper showing the effects on total factor productivity measures of substantial disaggregation into the quantities and prices paid for labour with various types of characteristics. Occupation, age, location, sex, race and education are characteristics that result in different prices per unit of labour time and different patterns of demand and or supply. Since data for the most detailed breakdown is only available in census years, two approaches seem sensible. First, Star and Hall (1973) have provided a very nice method of approximating a Divisia index of productivity when data is available at only the ends the beginning of a period. This would allow the very detailed census data to be used in productivity studies. Second, Statistics Canada could investigate the possibility of generating a time series on labour on a more disaggregated basis than currently available. This would undoubtedly involve some attempts at coordination with the Department of Labour and their occupational jungle. Finally, Statistics Canada will be prodded at some time to either produce or aid in the production of total factor productivity measures. At the moment this seems to be a treacherous area with any easy solution being confounded by the data. It might be possible to substantially improve the estimates if Statistics Canada could locate some three digit industries in which measures of capital utilization could be approximated. Two possible sources of information are important. First, changes in the number of shifts or the length of time open for business is an important indicator of the intensity of the use of the capital stock. Second, there used to be a practice of collecting information on electricity consumption in the census of manufacturing divided into lights and motors and other uses. The intensity of use of machinery and equipment can be approximated by relating consumption to the capacity of the stock of motors if information is available. Hopefully, Statistics Canada can make use of its unique capabilities as a source of data to permit a continuous improvement in the measurement of resource use.

Conclusions

Our major aim has been to indicate the usefulness of alternative measures of output. If producers' behaviour is to be studied in order to understand the effects of government policy then the technology relating gross outputs to primary and intermediate inputs must be studied. It should not be difficult for Statistics Canada to release gross output data for large segments of the economy since the detailed work on RDP already uses such data. The difficulties of providing data on intermediate inputs is undoubtedly more severe, but hopefully at least some aggregate measures of these can be prepared.

The currently available net production measures which were designed to fit into a consistent set of disaggregated accounts based on end products flowing to final demand are inadequate. They are useful for some descriptive purposes but do not allow a complete study of the economic behaviour of producers except under rather stringent conditions.

We have tried to provide some details of the problems that still have to be overcome in the measurement of outputs and inputs. It should be emphasized that the strong points of current Statistics Canada. practices have often been ignored since there are many interesting and complex problems remaining.

Appendix A: An Example, Cobb Douglas Production

The Cobb-Douglas production function has a long distinguished history in economics. It is a relatively easy form to manipulate and consequently we will use it here to illustrate several points concretely. Suppose that our three input production technology,
$Q = f(K,L,M)$ is Cobb-Douglas

$$Q = AK^{\alpha}L^{\beta}M^{\tau}$$

The Cobb-Douglas form is weakly separable and in fact is strongly separable since the elasticity of factor substitution is one between K, L and M (27). In addition, the form can obviously be written

$$Q = Bg(K,L) \cdot h(M)$$

where $g(K,L) = K^{\alpha}L^{\beta}$ and $h(M) = M^{\tau}$. Real value-added in this case is simply b $K^{\alpha}L^{\beta}$. In competitive equilibrium α and β will be the shares of total income paid to capital and labour. Real value added is thus a geometric mean of the primary inputs with weights that are the competitive shares.

Let us define value added, V, as

$$V = pQ\text{-}p_mM.p = \text{output price, } p_m = \text{materials price}$$

If in equilibrium, entrepreneurs are maximizing their profits, then the value of the marginal product of materials will be set equal to the materials price.

$$p.\partial Q/\partial M = p\tau AK^{\alpha}L^{\beta}M^{\tau-1} = \tau pQM^{-1} = p_m$$

Solving for M we have

$$M = \tau p.Q/p_m$$

and,

$$V = p.Q - \tau pQ = (1\text{-}\tau)pQ$$

Value added, for the Cobb-Douglas case, is proportional to the value of gross output. If we know that real value added VQ equals $K^{\alpha}L^{\beta}$, can we express this in terms of gross output and materials

$$VQ = K^{\alpha}L^{\beta} = Q.M^{-\tau}$$

Real value added is related to gross output and the quantity of materials but the relationship is such that there is no simple way of going from value added to real value added by deflating the former by a price index.

Suppose we consider defining a deflation procedure for calculating real value added. Two obvious candidates are (a) deflation by the output price, p, and (b) double deflation of outputs and inputs.

(a) <u>deflation by the output price</u>

$$VQ1 = Q(1-\tau)$$

Real value added is proportional to output. This procedure has been used in order to obtain an approximate measure of gross output when the latter is not available separately.

(b) <u>double deflation</u>

$$VQ2 = Q-M = Q(1-\tau p.p^{-1}m)$$

Double deflated real value added, VQ2, will not be proportional to gross output unless the relative price of output to materials is constant. Obviously the three measures of real output can move quite differently and the deflation of value added is not very helpful in measuring real value added even in this separable case.

Information on value added alone can be quite helpful in determining the technology. Suppose we do not have data on the materials used in production. In our simple Cobb-Douglas case, $V = (1-\tau)M$ and one can use information on value added to obtain estimates of M, if a Cobb-Douglas technology is assumed and the share of materials, τ, is known. Alternatively, it is possible to use the data on value added to estimate the production technology. This does not involve the assumption that one knows real value added only nominal value added. If we assume that materials are used competitively and substitute the quantity of materials into the production function we will obtain

$$Q = A_1 K^{\acute{o}} L^{\varepsilon} (p/p_m)^{\tilde{N}}$$

where, $A_1 = (A\tau^{\tau})^{1/1-\tau}$, ó = $\alpha/(1-\tau)$, $\varepsilon = ß/(1-\tau)$ and Ñ = $\tau/(1-\tau)$.

Now if gross output is not known but value added data are available then we can construct the following measure of value added

$$V = A_2 K^{ó} L^{\varepsilon} p^{░} p^{Ñ}{}_{m}$$

where ░ = $1/(1-\tau)$. This expression shows value added in terms of the primary inputs and the price of output and materials. It is this sort of equation that must be used in cases where information on gross output and materials is not provided. In general, value-added can be used in conjunction with prices of output and intermediate materials and primary input quantities. The general form will be

$$V = h(K,L,p,p_m)$$

This illustrates the possibilities of doing without information on the quantities of gross output and intermediate materials although it requires information on output prices and a material price index. Two points should be remembered. First, in most cases it is not possible to begin with an explicit production function and solve for an expression for V value added as we did with the Cobb-Douglas. However one can begin directly with an analytic expression for $V = h(K,L,p,p_m)$. Secondly alternative assumptions can be made about the use of materials. If the assumption is other than the value of the marginal product equals the factor cost than the exact properties of the new value added function will have to be specified. One obvious other extreme is that the flow of materials is fixed instead of being adjusted to recent prices because of long term contracts. Any adjustment will have to come through inventory models or else through the other inputs.

Appendix B: The Cambridge-Cambridge Capital Controversy

Within economics there has arisen a sharp controversy concerning the relationship between capital theory and the theory of production. We have basically ignored this debate in the main report but will comment briefly on it here. The recent book by Rymes (1971) and the paper by Read (1968) are an adequate sample of the literature for our purposes. Rymes for somewhat different reasons would certainly agree that information on gross output is needed to analyse production and that the net output measures currently produced are unsatisfactory particularly for productivity analysis. Rymes' discussion of output measurement is contained in Ch. 7 of his book (1971).

The disagreement amongst economists has never adequately been resolved and for the moment the neo-classical analysis used in this paper predominates in most areas of economics. The major disagreement does not centre on output but on the measurement of capital and the sense in which one can consider capital as a primary input. What the neo-Keynesian economists such as Rymes and Read desire is an alternative treatment of capital that focuses on the fact that capital is a commodity produced within the economy. What is required is a much more complex general equilibrium analysis of the different production processes. For example, Professor Read's suggestion in regard to productivity (1968, p. 352) "that the contribution of an intermediate good should not be assessed according to its output measure but according to its input measure" would require a simultaneous consideration of technical changes in all sectors.

While we do not ourselves agree with Professor Rymes or Mrs. Robinson and the Cambridge, England developments in regard to capital theory, this is not important for this report. I believe there would be substantial agreement on what data would be desirable although the uses to which we might put the data would substantially diverge.

Appendix C: Outputs: Production and Consumption

One of the confusions that appears to plague discussions of problem areas in measuring output such as the government and service sector is the distinction between production and consumption. Having had a brief exchange of views with the researchers in Statistics Canada and also having arrived at some of the same ideas somewhat independently, let me try and sketch a conceptual framework that I find useful. At the same time, I will attempt to integrate these ideas with the similar ideas which Mr. Bone sets out in his memo, "Towards a Theory of the Economics of the Service Industries".

Let us maintain the idea of utility function whose arguments are the commodities, goods, or services, which we are interested in measuring as output of the production technology. As we will argue below it does matter how we define the commodities. What we are suggesting is that we would like to identify the same commodity in the utility function and in the output of the production function. For the moment we can consider a utility function U(X) where X is a vector of outputs or commodities. Everyone agrees that utility is not measurable but participants in debates over the questions mentioned above tend to be inconsistent concerning the differences between welfare and production. Since we cannot measure utility we are not concerned with the joy, pain or ultimate effect on individual welfare that consuming a commodity may produce. We must settle for some description of a commodity that is measurable at least in theory and this can be quite some 'distance' from the ultimate, unmeasurable satisfaction. Three points seem important for the conceptual clarity of any discussion. First, the description of the commodity should not change over time. If it does the whole range of issues associated with quality change must be considered and we wish to avoid this for a moment. Second, the description of the commodity should be the same for the output of the production sector and the arguments of the utility function. If we consider a set of inputs being used to produce after a long change of events some ultimate satisfaction for a consumer and we must break this flow of events at some point, what should we do? In a private market economy, the chain is usually split at the point at which transactions are made. The producer has combined inputs to produce a commodity with given characteristics that the consumer purchases. The description of the characteristics should be the same so that the seller and buyer are transacting in a well-defined commodity. Let us assume that the set of transformations of inputs into final satisfaction does not change. The degree of 'packaging' is determined in the market place. If one wishes one can bring in the Becker-Lancaster theory of the transformation by consumers of purchased and own inputs into ultimate characteristics or satisfaction. In a sense the measurement of output is arbitrary since we accept the markets determination of the set of characteristics or attributes that are bought and sold. We attempt to quantify these transactions in terms of a definition of a commodity with a given bundle of characteristics and a price paid per commodity. In practice the feasible collection of price and quantity data for commodities finely distinguished by their characteristics is very limited. However, we want to first establish some conceptual clarity and then the problems of imperfect implementation can be considered. Please note that we will not consider implementation problems here. The above rather tortured description is necessary to clarify the ideas that the outputs of the production technology measured at the market boundary are identical to the commodities that are the arguments of the utility function. If the real world were 'kind' and there were no shifts in the market boundary, no changes in the producers" technology or the utility function then we could proceed to try and quantify the output and its value at the boundary. Remember that changes in the Becker-Lancaster consumers' technology are equivalent to changes in the utility function. Remaining in our artificial world of no change what special problems arise?

Two prominent cases have dominated many discussions in economics. First, what do we do if there is no market boundary where transactions take place? Second, can we always define a price and a quantity of output when there are market transactions at a boundary. Since we accept the market boundary as defining the characteristics of commodities when one exists, we believe that some degree of difficulty will always exist for the non-market cases. Many cases of non-market transactions from which we can infer a price. Alternatively, we may only wish to quantify the output, accepting the price as zero. The problem in this example spills over into our second case. How can we sensibly quantify an output in examples with or without market boundaries? The whole problem of measuring output in the commercial and non-commercial services is entered here. We believe that the distinction between goods and services is simply the fact that the former can more easily be quantified because the output has some physical or chemical dimensions that can be measured. Services inherently involve outputs for which there are no simple measurable dimensions. Two strong qualifications need to be made. First, industries that are classified as services produce outputs with characteristics which are a mixture of goods and services. Similarly, in many goods producing industries there is a substantial service component to the output. This suggests that all industries will have output imperfectly measured and that service industries might be approximated by some of the physical characteristics involved.

We have not stated anything about the purchasers' capabilities of knowing the characteristics of the commodities (goods or services) that they are buying. Implicitly they are assumed to know these as well as their utility function (or consumption technology). If the consumer is assumed to have a consumers' technology, then like any purchase of inputs the characteristics of the commodities he purchases will be derived from their capabilities to be transformed through the consumer technology into ultimate satisfactions. This implies that the set of characteristics of commodities that we wish to measure at whatever producer-consumer boundary is adopted is not arbitrary. Producers will attempt to respond to their perceptions of consumer demands and the latter will be linked through the consumers' technology to the 'ultimate satisfactions'. For this reason, in cases in which outputs are not well defined by a physical object, the characteristics that are suggested as output measures often are derived from inferences about what are the 'ultimate satisfactions'. It is not the case that one wishes to measure the ultimate satisfaction. It is the case that the characteristics of the commodity at the boundary are derived demands by the consumer and consequentially are tied to the 'ultimate satisfactions'. It will not be possible to ignore all welfare aspects in defining output or to measure physical characteristics of commodities unrelated to consumer demand. It makes sense to try and conceive of the set of characteristics that a consumer will want because otherwise what we will measure as output of the producing sector will not be the commodity bought by the consuming sector since the characteristics are not complete.

We may in non-commercial industries slice the continuum from input use to ultimate satisfaction at many different points. However, our argument is that wherever the boundaries we must be concerned with the same set of characteristics viewed from both consumers' and producers' viewpoints. This does not mean that we are confusing welfare and production but only states that the two are linked and we cannot ignore the links without errors.

Appendix D: Some Illustrations of Government Intermediate Products

The provision by the government of a system of justice and with public roads will illustrate the intermediate product nature of some government services. Suppose taxes are used to provide police court service and penal services to enforce the laws of a country. Many have argued that the present treatment of these expenditures in the national accounts is erroneous. For example, if the enforcement costs of a given level of safety for persons and real property increases, taxes might be raised and government expenditure increased in order to maintain a given safety level. There would be no change in NNP as currently measured since government expenditure would rise and some private expenditures fall due to lower disposable incomes. It can be argued that the costs of the justice system are what are sometimes called 'regrettable necessities'. That is, they are necessary costs of maintaining the well-being of persons and real property but do not add to welfare directly. Consequently, the increased costs of a given safety level reduces welfare and does not leave it unchanged. If the government did not respond to the increased costs of a given safety level, private individuals and firms might respond by directly increasing expenditures on private security devices or by allowing insurance rates to rise. In either case, the level of real output in the form of deliveries to final demand would fall since prices would rise and fewer factors would be available to produce private non-security goods. To the extent that households increased their expenditure or find their insurance rates increasing, then the current national accounts procedures would still show an increase in real NNP. If business firms respond the current procedures would show a decline in welfare.

The use of roads creates similar difficulties although the additional problems of congestion costs and the public goods nature of the output are perhaps more obvious in this case. First, if governments provide services to businesses or households without charge then ignoring congestion costs decisions on their use will be based on a zero market price. Consumers will use roads until an additional unit of road use has no benefit and businesses until the marginal product of extra road use is zero. Governments have usually had to impose weight, volume and size limits on business vehicles to hold down the free road use. The contribution of the free service input to business output will be reflected in the market value of the goods. This implies that if market valuations are to be used the market value of the government services at their cost or to increase the prices of the goods into which the free government intermediate services enter as inputs.

Consider some special cases of road use. Suppose that a particular road is only used by households to travel to cottages on weekends. If the road was built before the time period we are considering, then the expenditures for repair and maintenance are the only costs of road use that are entered as final products. The households perceive of the road as a free good as long as congestion costs are minimal. The households cost of a weekend at the cottage includes no road charge and the market value of the weekend should not include any costs of road use. If the resource value of the weekend were to be measured, then the costs of the road repair and maintenance as well as the imputed return on the capital tied up in the land and road should be included. Current practices measure neither market value nor resource cost. The same type of argument applies in the case of a road used extensively to transport goods by private business. Consumers pay the market price of goods delivered on public roads and this is the market value. The failure to price road use creates a difference between the market price and resource cost of the provision of goods including transportation. The national accounts fails to use a consistent valuation system on the one hand and includes intermediate goods as final products on the other.

Appendix E: Gross Output and Duplication

In implementing any conceptual framework for measuring production activity there is a practical problem associated with the boundaries at which outputs and inputs are to be measured. Conceptually the question is how to treat the own consumption or use of own produced output. There will be large differences in the quantity of gross output and materials inputs depending on the convention adopted for this. However for many purposes there is little or no difference in the usefulness of the information. Consider an extremely simple example. Gross output Q_G is produced using some of the produced output Q_M and labour L. The same homogenous commodity Q is an output and an input. The production technology can be written as

$$Q_G = f(Q_M, L)$$

OR

$$Q_N = Q_G - Q_M = g(L)$$

The net output formulation is the solution to the following maximization problem:

$$\text{maximize: } Q_N = Q_G - Q_M$$

$$\text{such that: } Q_G = f(Q_M, L)$$

$$L = L$$

The solution will be $Q_N = g(L)$ showing the maximum net output that can be produced with any given quantity of labour, L. Provided the input and output commodity are identical and we wish to assume that production units are efficient then either information on Q_G, Q_M, and L or Q_N and L will be equally useful. If production units do not use Q efficiently as an input then there will not be any well defined relationship between net output, Q_N, and labour input, L.

The example above is extremely simple but the arguments carry over to general cases. We can have many outputs and different types of labour capital and material inputs. It is still possible to derive a relationship between net outputs and inputs. This relationship depends on the efficient use of own production internal to the production unit. That is there will be a more complex maximization problem whose solution will be the relationship between net outputs and the inputs, excluding own produced and consumed products.

What implications does this have for the measurement of gross output. Let us assume for convenience that there is a smallest production unit called an establishment. We will not worry about the precise definition of an establishment but will simply assume that it is the smallest production unit from which accounting data on inputs and outputs can be collected. It is standard practice to measure gross outputs at this level as shipments (assume no inventories). This will automatically exclude any production for own use within the establishment. However when we group establishments together to form industries, at whatever aggregation level, there arises a problem. Suppose ten establishments are grouped together as an industry. The data collected at the establishment level includes transactions, i.e. shipments, between the ten establishments. If we simply add up the outputs and material inputs collected separately for the ten establishments and call the resulting output measure gross output are there any problems? No, one can argue that this is an adequate measure. Our earlier little example suggests that the important question is not whether the gross output measure includes or excludes the intra-industry shipments. The important fact is the consistency of the output and input measures. If output includes intra-industry shipments is equivalent to assuming that the industry is organized efficiently internally. If one does not wish to assume this then the grosser output concept and the materials flows are needed.

For practical reasons the gross concept is much easier to obtain. If one is considering a variety of subgroupings of industries a great deal of very detailed information is needed concerning the origin and destination of shipments by establishment for the net concept. It is unlikely that this will be available except at a fairly high level of aggregation or at unreasonable costs. There do not seem to be any severe costs to providing the grosser concepts at a substantially lower cost.

It might appear that we are being inconsistent. The establishment data are net of intra-establishment consumption of own output while intra-industry shipments are to be included in gross output. There is a literal inconsistency but I do not believe that it is important. In practice, as in theory the establishment is a creature of convenience. Whether a particular product is shipped from an establishment or is an intra-establishment is arbitrary. It depends on the accounting system, adopted by the firm. The same holds for the arbitrary SIC clarification. The groupings of establishment is based on some degree of output homogeneity. However due to the arbitrary nature of the establishment boundary the degree of input homogeneity is much less. While output homogeneity is desirable, it would be a pleasant surprise if this also yielded vertical homogeneity. Unfortunately, this does not occur in practice. The variety of market forces that confronts the establishment in addition to strictly engineering considerations will effect the range of activities that are internal to either an establishment or an industry.

In general, whatever output is consumed internal to a firm or an industry can be treated as part of both outputs and inputs or netted out. If the latter case is chosen then the assumption is being made that the internal allocation procedure is efficient. If one does not wish to make this assumption then the grosser series on inputs and outputs will be useful.

It is not correct to consider the grosser series as involving any duplication. We are trying to measure the outputs and inputs used in industries. If we are finally aggregating industry outputs to link with final demand expenditure aggregates then all outputs that do not flow to these users are eliminated.

Bibliography

Abramovitz, M. (1950), **Resource and Output Trends in the United States since 1870**, Occasional Paper 63, New York, NBER.

A Critique of the U.S. Income and Product Accounts, Studies in Income and Wealth, Volume XXII, NBER, Princeton University Press, Princeton, 1958.

Alterman, J. and Jacobs, E.E. (1961), "Estimates of Real Product in the United States by Industrial Sector, 1947-55", in *OIP*, pp. 275-314.

Arrow, K.J. (1972), "The Measurement of Real Value Added", Technical Report No. 60, IMSSS, Stanford University.

Berlinquette, V.R. (1954), "Measurement of Real Output", *Canadian Journal of Economics and Political Science.*

______ and Leacy, F.H. (1961), "The Estimation of Real Domestic Product by Final Expenditure and by Industry of Origin in Canada", in *OIP*, pp. 203-250.

Berndt, E.R. and Wood, D.O. (1974), "Technology, Prices, and the Derived Demand for Energy", Discussion Paper 74-9, University of British Columbia.

Brown, M. (1966), <u>On the Theory and Measurement of Technological Change</u>, Cambridge.

Bruno, M. "Value-Added Production Functions" in <u>An Econometric Approach to Production Theory</u>, D. McFadden, ed. Amsterdam, North-Holland, forthcoming.

Buchanan, J.M. and Forte, F. (1961), "The Evaluation of Public Services", <u>The Journal of Political Economy</u>, Volume LXIX, The University of Chicago Press.

Christensen, L.R. and Jorgenson, D.W. (1970), "U.S. Real Product and Real Factor Input, 1929-1967", <u>Review of Income and Wealth</u>, Series 16, pp. 19-50.

David, P.A. (1962), 'The Deflation of Value-Added', <u>Review of Economics and Statistics</u>, 44, pp. 148-55.

_____ (1966), 'Measuring Real Net Output: a Proposed Index', <u>Review of Economics and Statistics</u>, 48, pp. 419-25.

Denison, E.F. (1962), The Sources of Economic Growth in the United States and the Alternatives Before Us, Supplementary Paper No. 13, New York, Committee for Economic Development.

____ (1967), Why Growth Rates Differ: Post-war Experience in Nine Western Countries, Washington, D.C., The Brookings Institution.

Denny, M. (1972), Production Technologies: Intermediate Goods, Final Demands and Missing Information, unpublished manuscript.

Diewert, W.E. (1971), "An Application of the Sheppard Duality Theorem: a Generalized Leontief Production Function", Journal of Political Economy 79, pp. 481-507.

____, "Hicks Aggregation Theorem and the Existence of a Real Value-Added Function", Department of Manpower and Immigration, Ottawa, 1973.

____, "Homogeneous Work Separability and Exact Index Numbers, Report No. 122, IMSSS, Stanford University, 1974.

Fabricant, S. The Output of the Manufacturing Industries, 1899-1937, NBER, N.Y., 1940.

Fisher, F. (1969), "The Existence of Aggregate Production Functions, Econometrica, 37, pp. 553-57.

Garston, G.J. and Worton, D. (1968), "Problems in the Estimation of Industry Output in Current and Constant Dollars", in Kendrick (1968).

Geary, R.C. (1944), "The Concept of Net Volume of Output, with Special Reference to Irish Data", Journal of the Royal Statistical Society, N.S. 107, pp. 251-9.

Gehman, C. and Motheral, C. (1968), "Meausres of Industrial Production and Final Demand", in Kendrick (1968).

Gilbert, M. and Beckerman, W. (1961), "International Comparisons of Real Product and Productivity by Final Expenditures and by Industry" in OIP, pp. 251-274.

Gottsegen, J.J. and Ziemer, R.C. (1968), "Comparison of Federal Reserve and OBE Measures of Real Manufacturing Output, 1947-64", in Kendrick (1968).

Green, H.A.J. (1964), Aggregation in Economic Analysis: An Introductory Survey, Princeton: Princeton University Press.

Griliches, Z. and Jorgenson, D. (1966), "Sources of Measured Productivity Change: Capital Input", American Economic Review, Volume 56, pp. 50-61.

Griliches, Z. and Ringstad, V. (1971), Economies of Scale and the Form of the Production Function, North-Holland Publishing Company, Amsterdam.

Jaszi, G. (1958), "The Conceptual Basis of the Accounts: A Re-examination" in A. Critique...

Jones, R. (1965) "The Structure of Simple General Equilibrium Models", Journal of Political Economy.

Kendrick, J. (1968), "Measurement of Real Product", in A Critique of the United States Income and Product Accounts, NBER, Studies in Income and Wealth, 22, Princeton, pp. 405-425.

_____ (1961), Productivity Trends in the United States, Princeton University Press, Princeton.

_____ (ed.), The Industrial Composition of Income and Product, Studies in Income and Wealth, Volume XXXII, NBER, Columbia University Press, New York, 1968.

_____ (ed.), Total Investment, Capital and Economic Growth in the United States.

_____ (1972), Economic Accounts and Their Uses, McGraw-Hill, Inc., New York.

_____ Postwar Productivity Trends in the United States, 1948-69, NBER, General Series, New York, 1973.

Khang, C. (1971), "An Iro-value hours Involving Intermediate Goods and its' Applications to the Pure Theory of International Trade", Journal of Economic Theory, Vol. 1, p. 315-325.

Leontief, W. (1936), "Composite Commodities and the Problem of Index Numbers", Econometrica, r, pp. 39-59.

Marimont, Martin L. (1962), "G.N.P. by Major Industries", Survey of Current Business, United States Department of Commerce.

Nadiri, M.I. (1970), "Some Approaches to the Theory and Measurement of Total Factor Productivity: A Survey", Journal of Economic Literature, 8, pp. 1137-1177.

Nicholson, J.L. (1955), "National Income at Factor Cost or Market Prices?", The Economic Journal, Volume LXV, London.

_____ (1955), Some Problems in the Measurement of Real National Income, Income and Wealth Series IV, New York (NBER).

Output, Input and Productivity Measurement, Studies in Income and Wealth, Volume XXV, NBER, Princeton University Press, Princeton, 1961.

Pigou, A.C. (1972), The Economics of Welfare, 4th edition.

Problems in the International Comparison of Economic Accounts, Studies in Income and Wealth, Volume XX, NBER, Princeton University Press, Princeton, 1957.

Read, L.M. (1968), "The Measure of Total Factor Productivity Appropriate to Wage-Price Guidelines", Canadian Journal of Economics, 1, pp. 348-58.

Ruggles, N. and Ruggles, R. (1970), The Design of Economic Accounts, NBER, General Series, New York.

Rymes, T. (1971), On Concepts of Capital and Technical Change, Cambridge.

Salter, W.G. (1960), Productivity and Technological Change, Cambridge.

Samuelson, P.A. (1950), "Evaluation of Real National Income" Oxford Economic Papers, N.S. (2).

Siegal, I.H. (1961), "On the Design of Consistent Output and Input Indexes for Productivity Measurement" in Output Input and Productivity Measurement, NBER, Studies in Income and Wealth, 25, Princeton, 1961, pp. 23-46.

Sims, C. (1969), "Theoretical Basis for a Double Delated Index of Real Value-Added", Review of Economics and Statistics, 51, pp. 470-1.

Solow, R.M. (1957), "Technical Change and the Aggregate Production Function", Review of Economics and Statistics, Volume 39, pp. 312-320.

_____ (1960), "Investment and Technical Progress", in K.J. Arrow, S. Karlin, and P. Suppes (eds.), Mathematical Methods in the Social Sciences, 1959, Stanford, Stanford University Press, pp. 89-104.

Star, Spencer and Hall, Robert E. (1973), An Approximate Divisia Index of Total Factor Productivity, Discussion Paper 73-19, University of British Columbia.

Statistics Canada, Real Domestic Product by Industry, 1961 Base, Ottawa, Catalogue Number 61-506, Occasional Publication.

_____ Indexes of Real Domestic Product by Industry of Origin, 1963-61, Ottawa, Catalogue Number 61-505, Occasional Publication.

_____ Survey of Production System of National Accounts. Domestic Product by Industry, Ottawa, Catalogue Number 61-202, Annual.

_____ Indexes of Output Per Person Employed and Per Man-Hour in Canada, Commercial Non-agricultural Industris, 1947-63, Ottawa, Catalogue Number 14-501, Occasional Publication.

_____ Aggregate Productivity Trends, Ottawa, Catalogue Number 14-201, Annual.

_____ Indexes of Real Domestic Product by Industry, 1961-69, 1961-100, Ottawa, Catalogue Number 16-510, Occasional Publication.

_____ (1959), Revised Index of Industrial Production, 1935-1957, Ottawa, Catalogue Number 61-502.

_____ (1952), Revised Index of Industrial Production, 1935-1951, Ottawa, Reference Paper Number 34.

_____ (1956 and 1960), The Inter-Industry Flow of Goods and Service, Canada, 1949, (and Supplement), Ottawa, Catalogue Numbers 13-510 and 13-513.

_____ (1962), National Accounts Income and Expenditure, 1926-1956, Ottawa, Catalogue Number 13-502.

Stigler, G. (1961), "Economic Problems in Measuring Changes in Productivity", in OIP, pp. 47-78.

Stone, R. (1956), Quantity and Price Indexes in National Accounts, O.E.E.C., Paris.

United Nations, A System of National Accounts, Studies in Methods, Series F. No. 2, Rev. 3, New York, 1968.

U.S. Department of Commerce (1958), U.S. Income and Output.

_____ (1954), National Income - 1954 Edition.

_____ (1951), The Economic Accounts of the United States: Retrospect and Prospect, Survey of Current Business, Volume 51, Number 7, Part II.

_____ (1972), The Measurement of Productivity, Survey of Current Business, Volume 52, Number 5, Part II.

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ii. It is also true the discussions in the national accounting literature in particular are very vague. Summarizing them would require an extensive analysis of what they were saying in a more concrete form.

iii. The details of how to implement these concepts will not be investigated here. They will be discussed in later sections, when necessary.

iv. The inclusion of imports which we have been ignoring creates problems that are similar to those of material inputs at the industry level.

v. In general the imputed cost of using owned assets should exclude service charges associated with the transactions costs in asset or rental markets.

vi. Appendix D contains some brief examples of the difficulties with government intermediate outputs.

vii. It has been suggested that these experimental studies do not belong in Statistics Canada. However private researchers can only proceed in very limited ways without the active assistance of agencies such as Statistics Canada.

viii. This is the ratio of the marginal product of capital to the marginal product of labour.

ix. Since 1961 these are available monthly.

x. There is some information that actual practice is closely related to the methods discussed here.

xi. A brief and insightful summary is contained in Solow's article in Brown (8).

xii. Hopefully this will be available in the Canadian Journal of Economics with a year.