

Atlantic Association for Research in Mathematical Sciences  
Canadian Mathematical Society  
Memorial University of Newfoundland

ATLANTIC ALGEBRA CENTRE

# INFINITE DIMENSIONAL LIE ALGEBRAS

International Workshop

ORGANISING COMMITTEE:

YURI BAHTURIN

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*Jacobs University of Bremen, Germany*

## Schedule

## Abstracts of Talks

Bonne Bay Marine Station of MUN  
Norris Point, NL, Canada

*July 19 - 23, 2010*

MONDAY, JULY 19, 2010

- 9:00 - 9:30 **Registration/Opening of the workshop**  
9:30 - 10:30 **Ivan Penkov**  
*Jacobs University of Bremen, Germany*  
AN INTRODUCTION INTO LOCALLY FINITE LIE THEORY  
AND THE CORRESPONDING IND-GEOMETRY
- 10:30 - 11:00 **Coffee**
- 11:00 - 12:00 **Alexander Baranov**  
*University of Leicester, UK*  
INNER IDEALS OF SIMPLE LOCALLY FINITE LIE ALGEBRAS
- 12:00 - 2:00 **Lunch**
- 2:00 - 3:00 **Vera Serganova**  
*University of California - Berkeley, USA*  
ON INTEGRABLE MODULES OVER INDUCTIVE LIMITS  
OF SIMPLE LIE ALGEBRAS (joint work with I. Penkov)
- 3:05 - 4:05 **Vyacheslav Futorny**  
*University of São Paulo*  
TWISTED WEYL ALGEBRAS AND THEIR REPRESENTATIONS
- 4:05 - 4:30 **Coffee**
- 4:30 - 5:00 **Mark Colarusso**  
*Notre Dame University, USA*  
THE GELFAND-ZEITLIN INTEGRABLE SYSTEM
- 5:00 - 5:30 **Eugene Chibrikov**  
*Memorial University of Newfoundland*  
ON FREE SABININ ALGEBRAS

TUESDAY, JULY 20, 2010

- 9:30 - 10:30 **Helmut Strade**  
*Hamburg University, Germany*  
LOCALLY FINITE DIMENSIONAL SIMPLE LIE ALGEBRAS  
IN POSITIVE CHARACTERISTIC: INTERESTING OPEN PROBLEMS
- 10:30 - 11:00 **Coffee**
- 11:00 - 12:00 **Ivan Dimitrov**  
*Queen's University*  
WEIGHT MODULES OVER  $gl(\infty)$
- 12:00 - 2:00 **Lunch**
- 2:00 - 3:00 **Antonio Fernandez Lopez**  
*University of Malaga, Spain*  
A SHORT PROOF OF ZELMANOV'S THEOREM  
ON LIE ALGEBRAS WITH AN ALGEBRAIC ADJOINT REPRESENTATION
- 3:05 - 4:05 **Zhuang Niu**  
*Memorial University of Newfoundland*  
THE CLASSIFICATION OF SIMPLE AMENABLE  $C^*$ -ALGEBRAS. I
- 4:05 - 4:30 **Coffee**
- 4:30 - 5:00 **Zhuang Niu**  
*Memorial University of Newfoundland*  
THE CLASSIFICATION OF SIMPLE AMENABLE  $C^*$ -ALGEBRAS. II
- 5:00 - 5:30 **Elitza Gurova**  
*Jacobs University of Bremen, Germany*  
BOREL AND PARABOLIC SUBALGEBRAS  
OF SOME LOCALLY FINITE LIE ALGEBRAS

THURSDAY, JULY 22, 2010

- 9:30 - 10:30 **Georgia Benkart**  
*University of Wisconsin - Madison, USA*  
LIE ALGEBRAS WITH PRESCRIBED  $\mathfrak{sl}_n$  DECOMPOSITION
- 10:30 - 11:00 **Coffee**
- 11:00 - 12:00 **Siarhei Markouski**  
*Jacobs University of Bremen, Germany*  
ON HOMOMORPHISMS OF DIAGONAL LIE ALGEBRAS
- 12:00 - 2:00 **Lunch**
- 2:00 - 3:00 **Vera Serganova**  
*University of California - Berkeley, USA*  
TENSOR CATEGORY OVER  $\mathfrak{sl}(\infty)$ ,  $\mathfrak{o}(\infty)$  AND  $\mathfrak{sp}(\infty)$   
(joint work with E. Dan-Cohen and I. Penkov)
- 3:05 - 4:05 **Yuly Billig**  
*Carleton University, Ottawa*  
IRREDUCIBLE REPRESENTATIONS FOR THE  
LIE ALGEBRA OF VECTOR FIELDS ON A TORUS
- 4:05 - 4:30 **Coffee**
- 4:30 - 5:00 **Iryna Kashuba**  
*University of São Paulo*  
INDUCED MODULES FOR AFFINE KAC-MOODY ALGEBRAS
- 5:00 - 5:30 **Jason McGraw**  
*Memorial University of Newfoundland*  
GRADINGS ON THE SIMPLE CARTAN AND MELIKYAN TYPE LIE ALGEBRAS

FRIDAY, JULY 23, 2010

- 9:30 - 10:30 **Matej Brešar**  
*University of Ljubljana, Slovenia*  
FUNCTIONAL IDENTITIES  
AND THEIR APPLICATIONS TO LIE (SUPER)HOMOMORPHISMS
- 10:30 - 11:00 **Coffee**
- 11:00 - 12:00 **Ivan Dimitrov**  
*Queen's University*  
BOTT-BOREL-WEIL THEOREM FOR DIAGONAL IND-GROUPS
- 12:00 - 2:00 **Lunch**
- 2:00 - 3:00 **Esther Garcia**  
*Universidad Rey Juan Carlos, Madrid, Spain*  
and **Miguel Gomez Lozano**  
*Universidad de Malaga, Spain*  
A CHARACTERIZATION OF THE KOSTRIKIN RADICAL OF A LIE ALGEBRA
- 3:05 - 4:05 **Yuri Bahturin and Mikhail Kotchetov**  
*Memorial University of Newfoundland*  
GROUP GRADINGS ON LOCALLY FINITE LIE ALGEBRAS
- 4:05 - 4:30 **Coffee**
- 4:30 - 5:30 **Discussion. Open problems. Closure of the workshop**

## Yuri Bahturin and Mikhail Kotchetov

*Memorial University of Newfoundland*

### GROUP GRADINGS ON LOCALLY FINITE LIE ALGEBRAS

In this talk we are going to discuss recent results about the classification of group gradings by finitely generated abelian groups on locally finite associative and Lie algebras. Our methods include group schemes and functional identities.

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## Alexander Baranov

*University of Leicester, UK*

### INNER IDEALS OF SIMPLE LOCALLY FINITE LIE ALGEBRAS

A subspace  $I$  of a Lie algebra  $L$  is called an inner ideal if  $[I, [I, L]] \subset I$ . Inner ideals are closely related to ad-nilpotent elements (which proved to be useful in classification of the modular simple Lie algebras) and are helpful in constructing gradings in Lie algebras. Inner ideals were completely classified for finite dimensional simple Lie algebras and recently this classification was extended to the finitary simple Lie algebras by Lopez, Garcia and Lozano. We provide a complete description of inner ideals of any simple locally finite Lie algebra  $L$ . In particular, we show that  $L$  has a non-trivial inner ideal if and only if  $L$  is of diagonal type. If  $L$  is of diagonal type then we prove that inner ideals of  $L$  are obtained as intersections of left and right ideals of the diagonal enveloping algebra of  $L$ . As a corollary we obtain Lopez-Garcia-Lozano's classification for the finitary simple Lie algebras. This is a joint work with J.Rowley.

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## Georgia Benkart

*University of Wisconsin - Madison, USA*

### LIE ALGEBRAS WITH PRESCRIBED $\mathfrak{sl}_n$ DECOMPOSITION

This talk will describe Lie algebras that have a specific type of decomposition relative to an  $\mathfrak{sl}_n$  subalgebra. Examples will include certain direct limit Lie algebras and, via automorphisms of order 3, all the exceptional Lie algebras.

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**Yuly Billig**

*Carleton University, Ottawa*

IRREDUCIBLE REPRESENTATIONS FOR THE LIE ALGEBRA OF VECTOR FIELDS ON A  
TORUS

The goal of this work is to construct irreducible bounded weight modules for the Lie algebra of vector fields on a torus. These modules have a weight decomposition with finite-dimensional weight spaces and possess the property that the energy operator has spectrum bounded from below. We use generalized Wakimoto modules to give an explicit free-field realization of a family of such representations. The modules in this family are irreducible unless they belong to the chiral de Rham complex, introduced by Malikov, Schechtman and Vaintrob. This is a joint work with Vyacheslav Futorny.

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**Matej Bresar**

*University of Ljubljana, Slovenia*

FUNCTIONAL IDENTITIES AND THEIR APPLICATIONS TO LIE  
(SUPER)HOMOMORPHISMS

We will first give a brief survey of the theory of functional identities and its applications, particularly solutions of Herstein's problems on Lie homomorphisms between associative rings. A natural question whether these results can be extended to Lie superhomomorphisms appears. This seems plausible, but not easy. We will present the first results in this direction, obtained jointly with Yuri Bahturin and Špela Špenko.

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**Eugene Chibrikov**

*Memorial University of Newfoundland*

ON FREE SABININ ALGEBRAS

Sabinin algebras are algebraic objects that capture the local structure of analytic loops in the same way in which Lie algebras capture the local structure of Lie groups. They were introduced by L.Sabinin and P.Miheev in 1987.

In this report we discuss some recent results concerning free Sabinin algebras and construct a linear basis of a free Sabinin algebra which is a generalization of Shirshov's scheme for choosing bases of free Lie algebras.

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**Mark Colarusso**

*Notre Dame University, USA*

THE GELFAND-ZEITLIN INTEGRABLE SYSTEM

In the 1950's, Gelfand and Zeitlin produced a basis for finite dimensional highest weight representations for certain classical groups. Thirty years later, Guillemin and Sternberg produced an integrable system on conjugacy classes of Hermitian matrices that is related to the Gelfand-Zeitlin basis for the unitary group via geometric quantization. In 2006, Kostant and Wallach developed a complexified version of the Gelfand-Zeitlin integrable system on the full Lie algebra of  $n \times n$  complex matrices,  $\mathfrak{gl}(n, \mathbb{C})$ .

The Gelfand-Zeitlin system on  $\mathfrak{gl}(n, \mathbb{C})$  integrates to a holomorphic action of a group  $A \cong \mathbb{C}^{\frac{n(n-1)}{2}}$  on  $\mathfrak{gl}(n, \mathbb{C})$  whose orbits of dimension  $\frac{n(n-1)}{2}$  are Lagrangian submanifolds of regular adjoint orbits. In this talk, we will describe the orbit structure of the group  $A$  and discuss recent work with Sam Evens where we generalize a result of Kostant and Wallach concerning the algebraic integrability of the Gelfand-Zeitlin system to all  $A$ -orbits of dimension  $\frac{n(n-1)}{2}$ . We will also describe a non-linear version of the Gelfand-Zeitlin system for the dual Poisson Lie group  $GL(n, \mathbb{C})^*$ .

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**Ivan Dimitrov**

*Queen's University*

WEIGHT MODULES OVER  $\mathfrak{gl}(\infty)$

I will discuss the classification of irreducible weight modules with finite dimensional weight spaces over  $\mathfrak{gl}(\infty)$ . While methods and constructions from the theory of weight modules over simple finite dimensional Lie algebras apply, interesting phenomena specific to  $\mathfrak{gl}(\infty)$  arise. I will also discuss the classes of integrable modules and bounded modules.

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**Ivan Dimitrov**

*Queen's University*

BOTT-BOREL-WEIL THEOREM FOR DIAGONAL IND-GROUPS

I will present an analog of the Bott-Borel-Weil theorem for diagonal ind-groups. The main step in this work is the construction of an appropriate analog  $W$  of the Weyl group so that its action on weights is compatible with the Demazure "action" of the Weyl group on the cohomology of line bundles. Once  $W$  is defined, we prove that a statement analogous to the Bott-Borel-Weil theorem holds. This is a joint work with Ivan Penkov.

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# Antonio Fernandez Lopez

*Universidad de Malaga, Spain*

## A SHORT PROOF OF ZELMANOV'S THEOREM ON LIE ALGEBRAS WITH AN ALGEBRAIC ADJOINT REPRESENTATION

A celebrated theorem due to Zelmanov proves that a Lie algebra over a field of characteristic zero with an algebraic adjoint representation and satisfying a polynomial identity is locally finite-dimensional. In this note we give a shorter proof of this result. We replace Jordan pairs by Jordan algebra at a Jordan element, which fits much better in the transference between Lie and Jordan properties. Also, we use socle theory, the structure theorem of simple finitary Lie algebras, and the fact recently proved by E. Garcia and M. Gomez that any nondegenerate Lie algebra is a subdirect product of strongly prime Lie algebras, which allows us to reduce the proof to the strongly prime case, thus avoiding the difficulty in the original Zelmanov's proof of transferring primitive ideals of the Jordan pair defined by a finite grading to the Lie algebra.

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### Vyacheslav Futorny

*University of São Paulo, Brazil*

and

### Jonas T. Hartwig

*University of Copenhagen, Denmark*

## TWISTED WEYL ALGEBRAS AND THEIR REPRESENTATIONS

We will discuss the theory of twisted generalized Weyl algebras, a class of algebras generalizing classical Weyl algebras.

Let  $R$  be an algebra,  $\sigma_1, \dots, \sigma_n$  commuting algebra automorphisms of  $R$ ,  $t_1, \dots, t_n$  elements from the center of  $R$ , and  $\mu_{ij}$  an  $n \times n$  matrix of invertible scalars. To these data one associates a *twisted generalized Weyl algebra*  $\mathcal{A}_\mu(R, \sigma, t)$ , an associative  $\mathbb{Z}^n$ -graded algebra. These algebras were introduced by Mazorchuk and Turowska [9] and they are generalizations of the much studied generalized Weyl algebras, defined independently by Bavula [1], Jordan [5], and Rosenberg [11].

We introduce a new family of twisted generalized Weyl algebras, called multiparameter twisted Weyl algebras, for which we parametrize all simple quotients of a certain kind. Both Jordan's simple localization of the multiparameter quantized Weyl algebra and Hayashi's  $q$ -analog of the Weyl algebra are special cases of this construction.

The following embedding result can be viewed as a generalization of Bavula's embedding [1] of a generalized Weyl algebra into an euclidean ring.

**Theorem A 1** *Assume that*

$$\sigma_i \sigma_j(t_i t_j) = \mu_{ij} \mu_{ji} \sigma_i(t_i) \sigma_j(t_j), \quad \forall i, j = 1, \dots, n, i \neq j, \quad (0.1)$$

$$t_j \sigma_i \sigma_k(t_j) = \sigma_i(t_j) \sigma_k(t_j), \quad \forall i, j, k = 1, \dots, n, i \neq j \neq k \neq i. \quad (0.2)$$

*Then the following statements hold:*

- (a) *If  $t_1, \dots, t_n$  are invertible in  $R$ , then  $\mathcal{A}_\mu(R, \sigma, t)$  is graded isomorphic to a  $\mathbb{Z}^n$ -crossed product algebra over  $R$ .*

- (b) If  $t_1, \dots, t_n$  are regular in  $R$ , then  $\mathcal{A}_\mu(R, \sigma, t)$  can be embedded into a  $\mathbb{Z}^n$ -crossed product algebra over a localization of  $R$ .

We define a family of twisted generalized Weyl algebras, called *multiparameter twisted Weyl algebras*. This definition was inspired by an unpublished note by Benkart, where *multiparameter Weyl algebras* were introduced. These algebras are a particular case of the algebras of our class.

*Multiparameter quantized Weyl algebras*  $A_n^{\bar{q}, \Lambda}$  were introduced in [7] as a generalization of the quantized Weyl algebras obtained by Pusz and Woronowicz [10] in the context of quantum group covariant differential calculus. They are examples of twisted generalized Weyl algebras. Contrary to the usual Weyl algebras the algebra,  $A_n^{\bar{q}, \Lambda}$  is in general not simple, even for generic parameters. Jordan [6] found a certain natural simple localization of  $A_n^{\bar{q}, \Lambda}$ .

We have the following parametrization of simple quotients of multiparameter twisted Weyl algebras in terms of maximal ideals of certain Laurent polynomial rings. Jordan's localization of  $A_n^{\bar{q}, \Lambda}$  is an example in this family, as well as Hayashi's  $q$ -deformed Weyl algebras [4].

**Theorem B 1** Let  $A = A_n^k(r, s, \Lambda)$  be a multiparameter twisted Weyl algebra.

1. The assignment

$$\mathfrak{n} \mapsto A/\langle \mathfrak{n} \rangle \tag{0.3}$$

is a bijection between the set of maximal ideals in the invariant subring  $R^{\mathbb{Z}^n}$  and the set of simple quotients of  $A$  in which all  $X_i, Y_i$  ( $i = 1, \dots, n$ ) are regular.

2. For any  $\mathfrak{n} \in \text{Specm}(R^{\mathbb{Z}^n})$ , the quotient  $A/\langle \mathfrak{n} \rangle$  is isomorphic to the twisted generalized Weyl algebra  $\mathcal{A}_\mu(R/R\mathfrak{n}, \bar{\sigma}, \bar{t})$ , where  $\bar{\sigma}_g(r + R\mathfrak{n}) = \sigma_g(r) + R\mathfrak{n}$ ,  $\forall g \in \mathbb{Z}^n, r \in R$  and  $\bar{t}_i = t_i + R\mathfrak{n}$ ,  $\forall i$ .
3.  $A/\langle \mathfrak{n} \rangle$  is a domain for all  $\mathfrak{n} \in \text{Specm}(R^{\mathbb{Z}^n})$  if and only if  $\mathbb{Z}^{2n}/G$  is torsion-free, where  $G$  is the gradation group of  $R^{\mathbb{Z}^n}$ .

Simple weight modules over twisted generalized Weyl algebras have been studied in [9],[8],[3]. We classify all simple weight modules over any multiparameter twisted Weyl algebra. Extending results by Benkart and Ondrus [2], we also describe all Whittaker pairs up to isomorphism over a class of twisted generalized Weyl algebras which includes the multiparameter twisted Weyl algebras.

## References

- [1] V.V. Bavula, *Generalized Weyl algebras and their representations*, Algebra i Analiz **4** No. 1 (1992) 75–97; English transl. in St Petersburg Math. J. **4** (1993) 71–92.
- [2] G. Benkart, M. Ondrus, *Whittaker modules for generalized Weyl algebras*, Represent. Theory **13** (2009) 141–164.
- [3] J.T. Hartwig, *Locally finite simple weight modules over twisted generalized Weyl algebras*, J. Algebra **303** No. 1 (2006) 42–76.
- [4] T. Hayashi,  *$q$ -analogues of Clifford and Weyl algebras-spinor and oscillator representations of quantum enveloping algebras*, Comm. Math. Phys. **127** (1990) 129–144.
- [5] D. A. Jordan, *Primitivity in skew Laurent polynomial rings and related rings*, Math. Z. **213** (1993) 353–371.
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- [7] G. Maltsiniotis, *Groupes quantiques et structures différentielles*, C. R. Acad. Sci. Paris Sér. I Math. **311** No. 12 (1990) 831–834.
- [8] V. Mazorchuk, M. Ponomarenko, L. Turowska, *Some associative algebras related to  $U(\mathfrak{g})$  and twisted generalized Weyl algebras*, Math. Scand. **92** (2003) 5–30.

- [9] V. Mazorchuk, L. Turowska, *Simple weight modules over twisted generalized Weyl algebras*, Comm. Alg. **27** No. 6 (1999) 2613–2625.
- [10] W. Pusz, S. L. Woronowicz, *Twisted second quantization*, Reports on Math. Phys. **27** No. 2 (1989) 231–257.
- [11] A. L. Rosenberg, *Noncommutative algebraic geometry and representations of quantized algebras*, Kluwer, Dordrecht (1995).

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**Esther Garcia**  
*Universidad Rey Juan Carlos, Madrid, Spain*  
 and  
**Miguel Gomez Lozano**  
*Universidad de Malaga, Spain*

## A CHARACTERIZATION OF THE KOSTRIKIN RADICAL OF A LIE ALGEBRA

In this paper we study if the Kostrikin radical of a Lie algebra is the intersection of all its strongly prime ideals, and prove that this result is true for Lie algebras over fields of characteristic zero, for Lie algebras arising from associative algebras over rings of scalars with no 2-torsion, for Artinian Lie algebras over arbitrary rings of scalars, and for some others. In all these cases, this implies that nondegenerate Lie algebras are subdirect products of strongly prime Lie algebras, providing a structure theory for Lie algebras without any restriction on their dimension.

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**Elitza Gurova**  
*Jacobs University of Bremen, Germany*

## BOREL AND PARABOLIC SUBALGEBRAS OF SOME LOCALLY FINITE LIE ALGEBRAS

Let  $\mathfrak{g}$  be a locally finite Lie algebra of countable dimension. A Borel subalgebra of  $\mathfrak{g}$  is a maximal locally solvable subalgebra and a parabolic subalgebra is one that contains a Borel subalgebra. We are interested in describing all Borel and parabolic subalgebras of  $\mathfrak{g}$ . For the finitary Lie algebras  $gl(\infty)$ ,  $sl(\infty)$ ,  $so(\infty)$ , and  $sp(\infty)$  the problem is solved in a series of papers by I. Penkov, I. Dimitrov, and E. Dan-Cohen. In this talk we describe their results. We start with discussing the relationship between maximal toral and Borel subalgebras of  $gl(\infty)$ . We consider separately the case of a splitting Borel subalgebra, i.e. one that admits an exhaustion by finite-dimensional Borel subalgebras. We then introduce the notion of a generalized flag. The main result we discuss is that if  $\mathfrak{g}$  is one of  $gl(\infty)$  and  $sl(\infty)$  and  $V$  is the natural representation of  $\mathfrak{g}$ , then each Borel subalgebra of  $\mathfrak{g}$  is the stabilizer of a generalized flag in  $V$ , the uniqueness of which is implied by certain properties. We then generalize this statement to a statement for arbitrary parabolic subalgebras of  $gl(\infty)$ . The main difference with the Borel subalgebra case is that the description of general parabolic subalgebras has to use generalized flags in both the natural and the conatural representations. Finally, we make a short comparison with the case of maximal toral and Borel subalgebras of the Lie algebra  $gl(2^\infty)$ , which is a prototypical example of a diagonal but not finitary Lie algebra. This is a project I am working on jointly with E. Dan-Cohen.

**Iryna Kashuba**

*University of São Paulo, Brazil*

INDUCED MODULES FOR AFFINE KAC-MOODY ALGEBRAS

We will discuss induced modules of nonzero central charge with arbitrary multiplicities over affine Lie algebras. In particular, we consider modules induced from the so-called pseudo parabolic subalgebras. Given such a subalgebra  $P$  of an affine Lie algebra  $g$ , our main result establishes the equivalence between a certain category of  $P$ -induced  $g$ -modules and the category of weight  $P$ -modules with injective action of the central element of  $g$ . In particular, the induction functor preserves irreducible modules. If  $P$  is a parabolic subalgebra with a finite-dimensional Levi factor then it defines a unique pseudo parabolic subalgebra  $P(s)$  containing  $P$ . The structure of  $P$ -induced modules in this case is fully determined by the structure of  $P(s)$ -induced modules.

These results generalize similar reductions in particular cases previously considered by Futorny, Koenig and Mazorchuk, and also by Dimitrov, Futorny and Penkov. This is a joint talk with V. Futorny.

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**Sergei Markouski**

*Jacobs University of Bremen, Germany*

ON HOMOMORPHISMS OF DIAGONAL LIE ALGEBRAS

In the talk we discuss certain results on diagonal locally simple Lie algebras. The main part of these results is presented in the author's preprint "Locally simple subalgebras of diagonal Lie algebras", arxiv.org/abs/1002.1684.

Diagonal Lie algebras are defined as direct limits of finite-dimensional Lie algebras under diagonal injective homomorphisms. An explicit description of the isomorphism classes of diagonal locally simple Lie algebras is given in the paper [A. A. Baranov, A. G. Zhilinskii, Diagonal direct limits of simple Lie algebras, *Comm. Algebra*, 27 (1998), 2749-2766]. The three finitary infinite-dimensional Lie algebras  $sl(1)$ ,  $so(1)$ , and  $sp(1)$  are important special cases of diagonal locally simple Lie algebras. Many classical results have been extended to these three infinite-dimensional Lie algebras. In particular, in the paper [I. Dimitrov, I. Penkov, Locally semisimple and maximal subalgebras of the finitary Lie algebras  $gl(\infty)$ ,  $sl(\infty)$ ,  $so(\infty)$ , and  $sp(\infty)$ , *J. Algebra* 322 (2009), 2069-2081] all locally semisimple subalgebras of  $\mathfrak{g} \cong sl(\infty)$ ,  $so(\infty)$ , and  $sp(\infty)$  are described, and moreover all injective homomorphisms  $\mathfrak{s} \rightarrow \mathfrak{g}$  are described in terms of the action of  $\mathfrak{s}$  on the natural and the conatural  $\mathfrak{g}$ -modules. The preprint already mentioned makes a substantial contribution to further extending these results to the class of diagonal locally simple Lie algebras. In particular, all locally simple Lie subalgebras of any diagonal locally simple Lie algebra are described up to isomorphism. We discuss the theorem providing a list of conditions under which there exists an injective homomorphism  $\mathfrak{s} \rightarrow \mathfrak{g}$  of a locally simple Lie algebra  $\mathfrak{s}$  into a diagonal locally simple Lie algebra  $\mathfrak{g}$ .

Further, jointly with Ivan Penkov, we discuss certain invariants of homomorphisms of diagonal locally simple Lie algebras. The ideas and partial results on this topic may lead to a description of such homomorphisms in the future.

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**Jason McGraw**

*Memorial University of Newfoundland*

GRADINGS ON THE SIMPLE CARTAN AND MELIKYAN TYPE LIE ALGEBRAS

In this talk we explore the gradings by groups on the simple finite dimensional Cartan type Lie algebras and Melikyan algebras over algebraically closed fields of positive characteristic  $p > 2$  ( $p = 5$  for the Melikyan algebras). We approach the gradings by abelian groups without  $p$ -torsion on a simple Lie algebra  $L$  by looking at the dual group action. This action defines an abelian semisimple subgroup (quasi-torus) of the automorphism group of  $L$ . A result of Platonov says that any quasi-torus of an algebraic group is contained in the normalizer of a maximal torus. We show that if  $L$  is a simple graded Cartan or Melikyan type Lie algebra, then any quasi-torus of the automorphism group of  $L$  is contained in a maximal torus. Thus all gradings by groups without  $p$ -torsion are, up to isomorphism, coarsenings of the eigenspace decomposition of a maximal torus in the automorphism group.

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**Zhuang Niu**

*Memorial University of Newfoundland*

THE CLASSIFICATION OF SIMPLE AMENABLE  $C^*$ -ALGEBRAS

I will give a review on the recent progress on the classification of simple unital amenable  $C^*$ -algebras, especially on the classification of simple unital inductive limits of certain type I  $C^*$ -algebras.

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**Ivan Penkov**

*Jacobs University of Bremen, Germany*

AN INTRODUCTION INTO LOCALLY FINITE LIE THEORY AND THE CORRESPONDING IND-GEOMETRY

In this introductory lecture I will recall some basic notions within the theme of the workshop. I will put the structure theory of locally finite Lie algebras in the center and will relate it with representation theory and the geometry of homogeneous ind-spaces.

In particular, I plan to discuss some recent advances in the structure theory of finitary and diagonal Lie algebras and to present a general program of study of  $(\mathfrak{g}, \mathfrak{k})$ -modules. I will also briefly outline the current status of the geometric approach to the representation theory of locally finite Lie algebras. My talk can be considered as an introduction to the talks of Dimitrov, Gurova, Markouski, and Serganova.

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## Vera Serganova

*University of California - Berkeley, USA*

### ON INTEGRABLE MODULES OVER INDUCTIVE LIMITS OF SIMPLE LIE ALGEBRAS (joint work with I. Penkov)

We will discuss the category of integrable modules over inductive limits of simple Lie algebras. In particular, we will prove that this category has sufficiently many injective modules. Then we construct the subcategory which is closed under dualization and where all objects have finite Loewy length. We will prove that all simple objects in this category are tensor modules and that every object has a resolution by injective modules of finite length.

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## Vera Serganova

*University of California - Berkeley, USA*

### TENSOR CATEGORY OVER $\mathfrak{sl}(\infty)$ , $\mathfrak{o}(\infty)$ AND $\mathfrak{sp}(\infty)$ (joint work with E. Dan-Cohen and I. Penkov)

This is a continuation of the previous talk. We will prove that the category of tensor modules over  $\mathfrak{sl}(\infty)$ ,  $\mathfrak{o}(\infty)$  and  $\mathfrak{sp}(\infty)$  is Koszul and give a precise construction of the corresponding Koszul ring. As a consequence we will prove that the tensor categories of modules over  $\mathfrak{o}(\infty)$  and  $\mathfrak{sp}(\infty)$  are equivalent.

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## Helmut Strade

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### LOCALLY FINITE DIMENSIONAL SIMPLE LIE ALGEBRAS IN POSITIVE CHARACTERISTIC: INTERESTING OPEN PROBLEMS

In this talk we describe all simple finite dimensional Lie algebras over an algebraically closed field of characteristic  $p > 3$ . Due to the Block-Premet-Strade-Wilson classification these are the Lie algebras of classical, Cartan and Melikian type. In early work of Yu. A. Bahturin and the author these algebras have been used in two different constructions to build simple locally finite dimensional Lie algebras in positive characteristic. In the first construction the subalgebras with simple classical factors are bounded, in the second, conversely, the growth is determined by the growth of the classical factors. We will present quite a long list of problems arising from this context. In particular, we are interested in restrictedness, sandwich elements, natural maximal subalgebras, filtered deformations, and extensions of these constructions. Some of the problems are of more accessible and some of more challenging nature.

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