Atlantic Association for Research in the Mathematical Sciences
Fields Institute for Research in Mathematical Sciences
Memorial University of Newfoundland
Atlantic Algebra Centre
Network of Ontario Lie Theorists

Hopf Algebras and Algebraic Groups

International Workshop

June 13 - 17, 2016

Memorial University
St. John's, NL

Schedule and Abstracts of Talks

Registration and all lectures will be held in room SN-2109 of Science Building. The welcome reception will be held in room HH-3022 on June 12, 6–8 p.m.

Organizing Committee

Yuri Bahturin (Memorial University)
Mikhail Kochetov (Memorial University)
Yorck Sommerhäuser (Memorial University)
Kirill Zainoulline (University of Ottawa)

We thank all colleagues, students and staff at Memorial University and Fields Institute who helped us with organizing this workshop!

Monday, June 13, 2016

9:00	-	9:30	**Registration**
9:30	-	10:20	**Pavel Etingof** *Massachusetts Institute of Technology* Finite dimensional Hopf actions on division algebras and applications. I
10:30	-	11:00	**Coffee Break**
11:00	-	11:50	**Nicholas Andruskiewitsch** *Universidad Nacional de Córdoba* A finite-dimensional Lie algebra arising from a Nichols algebra of diagonal type
12:00	-	2:00	**Lunch Break**
2:00	-	2:50	**Vladislav Kharchenko** *Universidad Nacional Autónoma de México* Quantum Lie Theory
3:00	-	3:50	**Hans-Jürgen Schneider** *University of Munich* Nichols algebras and Nichols systems. I
4:00	-	4:30	**Coffee Break**
4:30	-	5:20	**Dmitri Nikshych** *University of New Hampshire* The Picard crossed module of a braided tensor category.
5:30	-	5:50	**Julia Plavnik** *Texas A&M University* Fermionic modular categories and the 16-fold way

Tuesday, June 14, 2016

9:30	-	10:20	**Pavel Etingof** *Massachusetts Institute of Technology* Finite dimensional Hopf actions on division algebras and applications. II
10:30	-	11:00	**Coffee Break**
11:00	-	11:50	**Alexander Merkurjev** *University of California at Los Angeles* Rationality problem for classifying spaces of algebraic groups
12:00	-	2:00	**Lunch Break**
2:00	-	2:50	**Nikita Karpenko** *University of Alberta* Chow ring of generically twisted varieties of complete flags
3:00	-	3:50	**Vladimir Popov** *Steklov Mathematical Institute* Coordinate algebras of connected affine algebraic groups: generators and relations
4:00	-	4:30	**Coffee Break**
4:30	-	4:50	**Marc-Antoine Leclerc** *University of Ottawa* Centroids of twisted forms of algebras
5:00	-	5:20	**Theo Raedschelders** *Vrije Universiteit Brussel* Universal Hopf algebras and highest weight categories
5:30	-	5:50	**Tiago Rodrigues-Macedo** *University of Ottawa* Automorphisms of ideals of polynomial rings

Thursday, June 16, 2016

9:30	-	10:20	**Hans-Jürgen Schneider** *University of Munich* Nichols algebras and Nichols systems. II
10:30	-	11:00	**Coffee Break**
11:00	-	11:50	**Nicholas Andruskiewitsch** *Universidad Nacional de Córdoba* Nichols algebras of finite Gelfand-Kirillov dimension over abelian groups
12:00	-	2:00	**Lunch Break**
2:00	-	2:50	**Susan Montgomery** *University of Southern California* Frobenius-Schur indicators for Hopf algebras and Fusion Categories
3:00	-	3:50	**Victor Ostrik** *University of Oregon* Symmetric tensor categories in positive characteristic
4:00	-	4:30	**Coffee Break**
4:30	-	5:20	**Hamid Usefi** *Memorial University of Newfoundland* Principal ideal restricted enveloping algebras
5:30	-	5:50	**Sergey Malev** *University of Edinburgh* Evaluations of non commutative polynomials on matrix algebras

Friday, June 17, 2016

9:30	-	10:20	**Vladislav Kharchenko** *Universidad Nacional Autónoma de México* Multilinear quantum Lie operations
10:30	-	11:00	**Coffee Break**
11:00	-	11:50	**Ivan Arzhantsev** *Higher School of Economics, Moscow* Additive actions on algebraic varieties
12:00	-	2:00	**Lunch Break**
2:00	-	2:50	**Mitja Mastnak** *Saint Mary's University* Liftings and bialgebra cohomology for coradically graded Hopf algebras
3:00	-	3:50	**Alexey Gordienko** *Vrije Universiteit Brussel* Hopf algebra actions and growth of polynomial H-identities
4:00	-	4:20	**Adrián Rodrigo-Escudero** *Universidad de Zaragoza* Gradings on real Lie algebras
4:30			**Closing**

Pavel Etingof

Massachusetts Institute of Technology

Finite dimensional Hopf actions on division algebras and applications

I will review a number of results on actions of a finite dimensional Hopf algebra H on fields and division algebras, and apply them to H-actions on various noncommutative algebras. In particular, I will explain how to prove that any H-action on the Weyl algebra and q-Weyl algebra (for generic enough q) in characteristic zero factors through a group action.
The proof uses reduction modulo prime powers and a result from algebraic number theory due to A. Perrucca. This is joint work with J. Cuadra and C. Walton.

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Nicholás Andruskiewitsch

Universidad Nacional de Córdoba

A finite-dimensional Lie algebra arising from a Nichols algebra of diagonal type

Let B be a finite-dimensional Nichols algebra of diagonal type over an algebraically closed field of characteristic 0. The distinguished pre-Nichols algebra of B, introduced and studied in [3], has several nice properties including finite GK-dimension and action of the Weyl groupoid.
Its graded dual, called the Lusztig algebra of B, was subsequently introduced and studied in [1]. We will outline these constructions. Then we will present the Lusztig algebra as an extension (as braided Hopf algebras) of B by the universal enveloping algebra of a graded nilpotent Lie algebra. We will exhibit explicitly this Lie algebra in several cases, including the all B of rank 2.

References:

[1] N. Andruskiewitsch, I. Angiono and F. Rossi Bertone. The divided powers algebra of a finite-dimensional Nichols algebra of diagonal type. Math. Res. Lett., to appear.

[2] N. Andruskiewitsch, I. Angiono and F. Rossi Bertone. A finite-dimensional Lie algebra arising from a Nichols algebra of diagonal type (rank 2). arXiv:1603.09387.

[3] I. Angiono. Distinguished Pre-Nichols algebras. Transf. Groups 21 (2016), 1-33.

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Vladislav Kharchenko

Universidad Nacional Autónoma de México

Quantum Lie Thoery

The numerous attempts over the previous 15-20 years to define a quantum Lie algebra as an elegant algebraic object with a binary "quantum" Lie bracket have not been widely accepted. In the talk we discuss an alternative approach that includes multivariable operations.

There are many fields in which multivariable operations replace the Lie bracket, such as investigations of skew derivations in ring theory, local analytic loop theory, and theoretical research on generalizations of Nambu mechanics.

Among the problems discussed in the talk are the following: multilinear quantum Lie operations, the principle generic quantum Lie operation, the basis of symmetric generic operations, Shestakov–Umirbaev operations for the Lie theory of nonassociative products. We also discuss extent to which a natural binary bracket may define somerthing that looks like a "quantum Lie algebra" inside of quantizations of Kac–Moody type in line with the PBW theorem for character Hopf algebras.

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Hans-Jürgen Schneider

University of Munich

Nichols algebras and Nichols systems

This is joint work with Istvan Heckenberger. I will introduce Nichols algebras of Yetter-Drinfeld modules over some Hopf algebra

with bijective antipode. The main problem of the talk is to understand the structure of the Nichols algebra when the Yetter-Drinfeld module M is a finite direct sum of simple Yetter-Drinfeld modules. In this case we define Nichols systems for M which generalize Nichols algebras and introduce reflection operators for Nichols systems. If the Weyl groupoid of M is finite, and if the Nichols system admits all reflections, then it is the Nichols algebra. As applications of this theorem we obtain rather conceptual proofs of two important known results: A finite-dimensional pointed Hopf algebra in characteristic zero with abelian group of group-like elements is generated by group-like and skew-primitive elements (Angiono); the + part of the deformed enveloping algebra of a semisimple Lie algebra is a Nichols algebra (Lusztig). I will sketch a proof of the theorem which is based on some general results on braided Hopf algebras in Yetter-Drinfeld categories, and on an abstract version of the Lusztig automorphisms of quantum groups. As part of the proof, a PBW-type decomposition of right coideal subalgebras in Nichols algebras is obtained.

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Dmitri Nikshych

University of New Hampshire

The Picard crossed module of a braided tensor category

To a braided finite tensor category C (e.g., to the category of representations of a finite dimensional Hopf algebra) one associates two groups: the group of braided autoequivalences of C and the Picard group of C consisting of invertible C-module categories. These groups are related to each other in a certain natural way: they form a crossed module (this algebraic gadget is the same thing as a categorical group). We discuss this structure called the Picard crossed module of C and its applications. We also explicitly describe Picard crossed modules of pointed braided tensor categories.
This is talk is based on a joint work with Alexei Davydov.

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Julia Plavnik

Texas A&M University

Fermionic modular categories and the 16-fold way

The study of spin and super- modular categories has been inspired, for example, by fermionic topological phases of matter and spin TQFT's. In this talk, we will introduce some basic definitions and properties and we will describe fermions for some specific examples. We will also present a construction called zesting and, if the time allows, we will give an idea of how to apply zesting to construct the minimal modular extensions of the super-modular categories $PSU(2)_{4m+2}$, $m \geq 0$.

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Alexander Merkurjev

University of California at Los Angeles

Rationality problem for classifying spaces of algebraic groups

Many classical objects in algebra such as simple algebras, quadratic forms, algebras with involutions, Cayley algebras etc, have large automorphism groups and can be studied by means of algebraic group theory. For example, for each type of algebraic objects there is an algebraic variety (called the classifying space of the corresponding algebraic group) that classifies the objects. The simpler the structure of this variety, the simpler the classification. For example, rationality of the classifying variety means that the objects can be described by algebraically independent parameters. In the lecture we will discuss the rationality property of classifying varieties.

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Nikita Karpenko

University of Alberta

Chow ring of generically twisted varieties of complete flags

Let G be a split simple affine algebraic group of type A or C over a field k, and let E be a standard generic G-torsor over a field extension of k. We compute the Chow ring of the variety of Borel subgroups of G (also called the variety of complete flags of G), twisted by E.
In most cases, the answer contains a large finite torsion subgroup.

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Vladimir L. Popov

Steklov Mathematical Institute of Russian Academy of Sciences

Coordinate algebras of connected affine algebraic groups: generators and relations

For the coordinate algebras of abelian varieties, the problem of finding a presentation by generators and relations canonically determined by the group structure has been explored and solved by D. Mumford in 1966. The analogous problem for connected affine algebraic groups naturally arises. The talk is intended to describe its solution based on solving two problems posed by D. E. Flath and J. Towber in 1992. From the standpoint of this theory, the usual naive presentation of $SL(n)$ as a hypersurface det $= 1$ in an n^2-dimensional affine space is adequate only for $n = 2$: the canonical presentation defines $SL(3)$ as the intersection of 2 homogeneous and 2 inhomogeneous quadrics in a 12-dimensional affine space, $SL(4)$ as the intersection of 20 homogeneous and 3 inhomogeneous quadrics in a 28-dimensional affine space, etc.

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Marc-Antoine Leclerc

University of Ottawa

Centroids of twisted forms of algebras

We define the centroid of dimodules and discuss their properties. In particular, we describe the centroid of twisted forms of algebras. This is joint work with Erhard Neher.

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Tiago Rodrigues-Macedo

University of Ottawa

Automorphisms of ideals of polynomial rings

Let R be a commutative integral domain with unit, f be a non-constant monic polynomial in $R[t]$, and $I_f \subset R[t]$ be the ideal generated by f. In this paper we study the group of R-algebra automorphisms of the R-algebra without unit I_f. We show that, if f has only one root (possibly with multiplicity), then $\mathrm{Aut}(I_f) \cong R^\times$. We also show that, under certain mild hypotheses, if f has at least two different roots in the algebraic closure of the quotient field of R, then $\mathrm{Aut}(I_f)$ is a cyclic group and its order can be completely determined by analyzing the roots of f. This is a joint work with T. de Mello.

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Theo Raedschelders

Vrije Universiteit Brussel

Universal Hopf algebras and highest weight categories

For any Koszul Artin-Schelter regular algebra A, we consider a version of the universal Hopf algebra $\mathrm{aut}(A)$ coacting on A, introduced by Manin. To study the representations (i.e., finite dimensional comodules) of this Hopf algebra, we use the Tannaka-Krein formalism. Specifically, we construct an explicit combinatorial rigid monoidal category U, equipped with a functor M to finite dimensional vector spaces such that $\mathrm{aut}(A) = \mathrm{coend}(M)$. Using this pair (U, M) we show that $\mathrm{aut}(A)$ is quasi-hereditary as a coalgebra and in addition is derived equivalent to the representation category of U.

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Alexander Merkurjev

University of California at Los Angeles

From simple algebras to the Bloch-Kato conjecture

Hamilton quaternion algebra over the reals was the first nontrivial example of a simple algebra discovered in 1843. I will review the history of the study of simple algebras over arbitrary fields that was culminated in the proof of the Bloch-Kato Conjecture by Voevodsky.

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Nicholás Andruskiewitsch

Universidad Nacional de Córdoba

Nichols algebras of finite Gelfand-Kirillov dimension over abelian groups

I will give an overview of the classification of Nichols algebras of finite GK-dimension over abelian groups, including numerous new examples.

References:

[1] N. Andruskiewitsch, I. Angiono and I. Heckenberger. On finite GK-dimensional Nichols algebras over abelian groups, in preparation.

[2] N. Andruskiewitsch, I. Angiono and I. Heckenberger. Liftings of Jordan and super Jordan planes. arXiv:1512.09271.

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Susan Montgomery

University of Southern California

Frobenius-Schur indicators for Hopf algebras and Fusion Categories

We survey recent results on the values of the FS indicators for specific Hopf algebras, such as bismash products, and for certain near groups; these are fusion categories arising in von Neumann algebras.

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Victor Ostrik

University of Oregon

Symmetric tensor categories in positive characteristic

A well known Delignes theorem states that over an algebraically closed field of characteristic zero a symmetric tensor category satisfying suitable finiteness assumptions admits a tensor functor to the category of super vector spaces. This implies that any such category can be described in terms of representations of a suitable super group. Examples constructed by Gelfand and Kazhdan show that this result does not hold over fields of positive characteristic. In this talk I will propose a conjecture describing what happens in the positive characteristic case. Our main result is that this conjecture holds for semisimple categories with finitely many simple objects.

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Hamid Usefi

Memorial University of Newfoundland

Principal ideal restricted enveloping algebras

Let L be a restricted Lie algebra over a field of positive characteristic. We prove that the restricted enveloping algebra of L is a principal ideal ring if and only if L is an extension of a finite-dimensional torus by a cyclic restricted Lie algebra. We shall discuss an analogue of this question in the Hopf algebra setting and formulate a problem for future research.

Joint work with Salvatore Siciliano.

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Sergey Malev

University of Edinburgh

Evaluations of non commutative polynomials on matrix algebras

Let p be a multilinear polynomial in several non-commuting variables with coefficients in an arbitrary field K. Kaplansky conjectured that for any n, the image of p evaluated on the set $M_n(K)$ of n by n matrices is either zero, or the set of scalar matrices, or the set $sl_n(K)$ of matrices of trace 0, or all of $M_n(K)$. We will present results related to Kaplansky conjecture in cases $n = 2$ and $n = 3$. We will also present a classification of possible images of multilinear polynomials evaluated on quaternion algebra.

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Ivan Arzhantsev

Higher School of Economics, Moscow, Russia

Additive actions on algebraic varieties

By an additive action on a complex algebraic variety X of dimension n we mean a regular action $G_a^n \times X \to X$ with an open orbit of the commutative unipotent group G_a^n. We begin with a survey of results on additive actions including Hassett-Tschinkel's correspondence for projective spaces and the case of (generalized) flag varieties.
We prove that if a complete toric variety X admits an additive action, then it admits an additive action normalized by the acting torus. Normalized additive actions on a toric variety X are in bijection with complete collections of Demazure roots of the fan of X. Moreover, any two normalized additive actions on X are isomorphic. The talk is based on a joint work with Elena Romaskevich.

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Mitja Mastnak

Saint Mary's University

Liftings and bialgebra cohomology for coradically graded Hopf algebras

In the talk I will survey some tools for computing the truncated Gerstenhaber-Schack bialgebra cohomology for some classes of Hopf algebras. I will explain how to interpret the second cohomology group as the infinitesimal parts of formal graded deformations and how formal graded deformations corespond to liftings. I will present some examples from the joint work with G. Garcia and F. Fantino (in progress). Joint work with I. Angiono and M. Kochetov regarding the rigidity of Nichols algebras will also be mentioned.

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Alexey Gordienko

Vrije Universiteit Brussel

Hopf algebra actions and growth of polynomial H-identities

H-module algebras, where H is a Hopf algebra, can be studied from different points of view. In the talk we discuss the structure of H-simple algebras, i.e. algebras that do not have non-trivial ideals that are H-submodules at the same time. This study can be considered as a natural continuation of the study of graded-simple group graded algebras. Our particular interest to this topic is concerned with polynomial H-identities and, more precisely, with the H-module analog of S. A. Amitsur conjecture. In the case of finite dimensional H-module algebras this conjecture can be derived from the existence for any H-simple algebra B of an H-polynomial f with many alternations that is not a polynomial H-identity of B. (Denote this property by (*).) Such an H-polynomial always exists if B is semisimple in the ordinary sense. However, if H itself is not semisimple, the Jacobson radical of B is not necessarily an H-submodule and can be non-trivial.

Many pointed (and not only pointed) Hopf algebras H can be obtained from smaller algebras H_1 using Ore extensions by adding

to H_1 a skew-primitive element. It turns out that study of H-actions can be largely reduced to the study of H_1-actions. In particular, we will show how the analog of Amitsur's conjecture for polynomial H-identities can be reduced to the same problem for H_1. This enable us to prove the conjecture for a wide class of Hopf algebras H.

Also we will discuss the problem of classification of H-simple algebras.

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Adrián Rodrigo-Escudero

Universidad de Zaragoza, visiting Memorial University

Gradings on real Lie algebras

Gradings on Lie algebras are in one-to-one correspondence with gradings on associative algebras and involutions that are compatible with the grading. First I am going to speak about the recent classification of gradings on associative real algebras, especially about my contribution (Linear Algebra and its Applications, Volume 493, Pages 164-182): the particular case when the grading is a division grading. Also, I am going to give some results about involutions on such graded algebras, which is what I am studying now.

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