Atlantic Association for Research in Mathematical Sciences
Memorial University of Newfoundland
Atlantic Algebra Centre

International Workshop

Graded Algebras & Superalgebras

August 29 - September 2, 2008

Schedule and Abstracts of Presentations

Memorial University of Newfoundland

St. John's, Newfoundland, Canada

Rooms

All lectures and talks on August 29 - September 1 will be held in Lecture Hall AA-1043 (Arts and Administration Building). On September 2 lectures and talks will be in Lecture Hall C-2045 (Chemistry Building).

Morning Coffee and Coffee breaks will be in Coffee Room of the Department of Mathematics and Statistics HH-3022 (Henrietta Harvey Building).

Registration will be in the office of Ms Ros English HH-3002.

You are always welcome to visit the office of Atlantic Algebra Centre at HH-2010.

Friday, August 29, 2008

8:30 - 9:00	HH-3002	**Registration**
9:00 - 9:30	HH-3022	*Coffee*/**Opening** Meeting Dr Eddy Campbell, Acting President Memorial University of Newfoundland
9:30 - 10:20	AA-1043	**Georgia Benkart** *University of Wisconsin - Madison, USA* Weyl-like algebras and their modules
10:30 - 11:20	AA-1043	**Dimitry Leites** *Stockholm University, Sweden, also ASSMS, HEC, Lahore* Classification of simple finite dimensional modular Lie superalgebras with Cartan matrix
11:20 - 11:40	HH-3022	*Coffee*
11:40 - 12:30	AA-1043	**Alberto Elduque** *University of Zaragoza, Spain* Gradings on composition algebras
		L U N C H
2:30 - 3:20	AA-1043	**Erhard Neher** *University of Ottawa* Universal central extensions of graded Lie algebras
3:30 - 4:10	AA-1043	**Yoji Yoshii** *Akita National College of Technology, Japan* Locally affine root systems and locally affine Lie algebras
4:10 - 4:30	HH-3022	*Coffee*
4:30 - 5:10	AA-1043	**Cristina Draper** *University of Malaga, Spain* Some features and techniques on the gradings on exceptional Lie algebras

Saturday, August 30, 2008

9:00 - 9:30	HH-3022	*Coffee*
9:30 - 10:20	AA-1043	**Ivan Shestakov** *University of Sao Paulo, Brazil* The Coordinatization Theorem for Jordan Algebras and Superalgebras
10:30 - 11:20	AA-1043	**Vesselin Drensky** *Bulgarian Academy of Sciences* Graded algebra structure of the centralizer of a matrix
11:20 - 11:40	HH-3022	*Coffee*
11:40 - 12:30	AA-1043	**Don Passman** *University of Wisconsin - Madison, USA* Filtrations in semisimple rings and Lie algebras
		L U N C H
2:30 - 3:20	AA-1043	**Eli Aljadeff** *Technion, Israel* On G-fine gradings on matrix algebras and their polynomial identities
3:30 - 4:10	AA-1043	**Vincenzo Nardozza** *University of Bari, Italy* Graded algebras and tensor product theorems
4:10 - 4:30	HH-3022	*Coffee*
4:30 - 5:10	AA-1043	**Michael Natapov** *Indiana University - Bloomington, USA* On graded polynomial identities of matrices

Monday, September 1, 2008

9:00 - 9:30	HH-3022	*Coffee*
9:30 - 10:20	AA-1043	**Georgia Benkart** *University of Wisconsin - Madison, USA* Centralizers of Groups of Matrices
10:30 - 11:20	AA-1043	**Dimitry Leites** *Stockholm University, Sweden, also ASSMS, HEC, Lahore* Towards classification of simple finite dimensional modular Lie superalgebras
11:20 - 11:40	HH-3022	*Coffee*
11:40 - 12:30	AA-1043	**Oleg Smirnov** *College of Charleston, USA* On Classification of simple Kantor pairs
		L U N C H
2:30 - 3:20	AA-1043	**Matej Bresar** *University of Maribor, Slovenia* Functional identities and their application to graded algebras
3:30 - 4:10	AA-1043	**Deanna Caveney** *College of Charleston, USA* Universal Imbeddings of Jordan systems into graded Lie algebras
4:10 - 4:30	HH-3022	*Coffee*
4:30 - 5:10	AA-1043	**Marina Tvalavadze** *University of Saskatchewan* On varieties parameterizing graded complex Lie algebras

Tuesday, September 2, 2008

9:00 - 9:30	HH-3022	*Coffee*
9:30 - 10:20	C-2045	**Ivan Shestakov** *University of Sao Paulo, Brazil* The skew-symmetric identities of octonions
10:30 - 11:20	C-2045	**Vesselin Drensky** *Bulgarian Academy of Sciences* Computing with multiple symmetric functions and graded cocharacters of graded PI-algebras
11:20 - 11:50	HH-3022	*Coffee*
11:50 - 12:40	C-2045	**Plamen Koshlukov** *University of Campinas, Brazil* Identities in graded tensor products of PI algebras
		L U N C H
2:40 - 3:30	C-2045	**Mikhail Kochetov** *Memorial University of Newfoundland* Formal group actions and gradings by p-groups in characteristic p
3:40 - 4:20	C-2045	**Jason McGraw** *Memorial University of Newfoundland* Gradings on simple Lie algebras of Cartan type
		Closure of the workshop

E. Aljadeff

Technion, Israel

On G-fine gradings on matrix algebras and their polynomial identities

It is known that any G-grading on $M_n(\mathbb{C})$ is given by a suitable combination of two basic type of gradings, fine and elementary (Bahturin and Zaicev). In this lecture I'll consider fine gradings and their polynomial identities. For such a grading we construct the corresponding universal G-graded algebra U. In particular we give a precise list of groups G for which the central localization of U is a division algebra. Important parts of this construction are extended to a much more general context, namely to H-comodule algebras where H is a finite dimensional Hopf algebra. (Joint work with Haile and Natapov (the G-case), Kassel (the H-case)).

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Georgia Benkart

University of Wisconsin-Madison, USA

Weyl-like Algebras and Their Modules

Weyl algebras have appeared in numerous settings in mathematics and physics. They are unital associative algebras with generators x_i, y_i and defining relations

$$\begin{aligned} [y_i, x_j] &= \delta_{i,j}1 \\ [x_i, x_j] &= 0 = [y_i, y_j] \end{aligned}$$

where $[a, b] = ab - ba$. Many graded associative algebras such as the universal enveloping algebra of sl_2 and its various quantum analogues, quantum planes, down-up algebras, etc. are sufficiently Weyl-like that their modules can be studied using the same techniques that apply for Weyl algebras. This one-approach-fits-all talk will survey some methods that can be used to investigate modules for Weyl-like algebras over arbitrary fields.

Georgia Benkart

University of Wisconsin-Madison, USA

Centralizers of Groups of Matrices

Issai Schur's 1901 dissertation on the representations of the general linear group GL_n of invertible n by n matrices was the thesis that launched a thousand papers. It has influenced work on symmetric functions, diagram algebras, knot and link invariants, and much more. The key ingredient in Schur's investigations was the centralizer algebra of the GL_n-action. In this talk we will explore GL_n-actions on various spaces, their centralizer algebras, and the resulting combinatorics.

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Sofiane Bouarroudj[1], Pavel Grozman[2], Dimitry Leites[3]

[1] *United Arab Emirates University, United Arab Emirates;* [2] *EquaSimulation AB, Sweden;* [3] *Abdus Salam School of Mathematical Sciences, Pakistan*

Classification of simple finite dimensional modular Lie superalgebras with Cartan matrix

Finite dimensional modular Lie superalgebras over algebraically closed fields with indecomposable Cartan matrices are classified under some technical hypotheses. If the Cartan matrix is invertible, the corresponding Lie superalgebra is simple otherwise the quotients of the derived Lie superalgebra modulo center is simple (if its rank is greater than 1). Twelve new exceptional simple modular Lie superalgebras are discovered.

Several features of classic notions or notions themselves are clarified or introduced, e.g., Cartan matrix, restrictedness, Dynkin diagram, Chevalley generators, and even Lie superalgebra if the characteristic is equal to 2. Interesting phenomena in characteristic 2:

(1) all simple Lie superalgebras with Cartan matrix are obtained from simple Lie algebras with Cartan matrix by declaring several (any) of its Chevalley generators odd;

(2) there is a simple Lie superalgebra with a solvable even part.

Matej Brešar

University of Maribor, Slovenia

Functional identities and their applications to graded algebras

We will first give a brief survey on the theory of functional identities, and then present two recent applications obtained jointly with Yuri Bahturin. The first one concerns group gradings on Lie algebras, and the second one concerns automorphisms of Lie superalgebras.

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D.M. Caveny, O.N. Smirnov

College of Charleston, USA

Universal Imbeddings of Jordan systems into graded Lie algebras

In the early 60's, Tits, Kantor, and Koecher developed a construction that produced a Lie algebra, TKK(J), from a Jordan algebra J. Working with a modification of the TKK construction that has functorial properties, we establish an equivalence between a category of Jordan algebras and a category of graded Lie algebras. We also give a homological characterization of this class of Lie algebras.

Cristina Draper Fontanals

University of Malaga, Spain

Some features and techniques on gradings on exceptional Lie algebras

(Part of work jointly with several coauthors: A. Calderón, A. Elduque, C. Martín and A. Viruel)

Among simple Lie algebras, the exceptional ones are remarkable by appearing in a range of situations and by their sometimes unexpected behavior. Gradings on them are related to different ways of looking at and understanding them.

The work [10] states that all of these gradings are group gradings. Although [5] realizes that the proof is false, everything seems to indicate that every grading on a simple Lie algebra is in fact a group grading. The existence of the group makes it possible to use the techniques of algebraic groups in the matter, because the problem can be stated in terms of simultaneous diagonalizable automorphisms.

Since the exhaustive description of gradings is a nightmare, it is usual to restrict the study to fine gradings. Fine gradings summarize the different ways of splitting an algebra. All the information is compressed in fine gradings because any other group grading can be obtained by putting together pieces of such fine grading. Besides the most known fine grading, the root decomposition, has proved to be a powerful tool that geometricians, physicists and algebraists have employed. This has been an extra motivation to study other possible fine gradings, a problem posed in [10]. This problem is equivalent to find the maximal diagonalizable groups of the automorphism group of the Lie algebra under study.

This talk is focused to explore different ways of finding (group) gradings, so as to remark which things can be expected to happen. We will do a brief tour around the four exceptional Lie algebras of the smallest dimensions, the algebras $\mathfrak{g}_2$, $\mathfrak{f}_4$, $\mathfrak{d}_4$ and $\mathfrak{e}_6$, to describe not only which are the fine gradings (and quick ways to describe them), but specially to explain which techniques have worked in the different cases.

All the possible gradings on simple Lie algebras of type A_l had been described in [3], and of type B_l, C_l and D_l ($l \neq 4$) in [1], but the used techniques are related to the associative algebras and hence are not valid for the exceptional cases.

As $\mathfrak{g}_2$ is the set of derivations on an octonion algebra $\mathcal{C}$, and the automorphism groups of $\mathcal{C}$ and $\mathfrak{g}_2$ are isomorphic groups, any grading of $\mathfrak{g}_2$ will be induced by a grading on $\mathcal{C}$. The same occurs when considering $\mathfrak{f}_4$ and the Albert algebra $H_3(\mathcal{C})$, whose automorphism groups are also related by means of the adjoint map. This mechanism for passing gradings provides some information, but not enough because it does not preserve equivalent gradings.

In the case of $\mathfrak{g}_2$, some simple tools, as the universal group, are enough to obtain all the fine gradings, only one different from the root decomposition (see

[6] or [2]), but do not suffice to work in $\mathfrak{f}_4$. The use of other algebraic models of $\mathfrak{f}_4$ and $H_3(\mathcal{C})$ allow us to describe all the gradings, but in order to prove that the description is complete we will go through the algebraic groups. The key fact is that each grading is produced by a quasitorus of the group of automorphisms of the algebra, but these quasitori live inside the normalizer of a maximal torus, which can be implemented in the computer. Working in the normalizer is a technical tool, but very powerful, which allows us to describe completely the nontoral gradings, so as the fine gradings, on $\mathfrak{f}_4$.

In $\mathfrak{d}_4$, there are only 3 fine gradings involving outer automorphisms. One way to aboard this problem is to know the centralizers of the outer automorphisms, since these arguments reduce the rank of the algebra under study. That is, to use induction arguments.

Finally, in $\mathfrak{e}_6$, we have combined the induction arguments, the computational methods and the results about centralizers known by geometricians, to get the fine gradings. To describe them, it suffices to use models based upon linear algebra.

References

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Vesselin Drensky

Bulgarian Academy of Sciences, Bulgaria

Computing with multiple symmetric functions and graded cocharacters of graded PI-algebras

This gives an easy approach (including effective algorithms) how to find the cocharacters of the G-graded identities (or $*$-cocharacters for algebras with involution) if the Hilbert series of the relatively free algebra is known. The method uses some old results (around 1900) on the number of positive solutions of linear Diophantine equations. The methods can be also used for invariant theory of matrices under the action of classical groups. (This is based on joint work with several people - Georgy Genov, Plamen Koev, Silvia Boumova.)

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Vesselin Drensky

Bulgarian Academy of Sciences, Bulgaria

Graded algebra structure of the centralizer of a matrix

Recent results of Nelson and Ton-That and of Szigeti and van Wyk describe the centralizer of an $n \times n$ matrix over any field. Combining these results, it has turned out that the centralizer has a structure of a graded finite-dimensional algebra. We give a very simple explicit form of the centralizer. It has turned out that such algebras appear in the description of polynomial identities and superidentities of algebras satisfying the identities of 2×2 matrix algebras and other PI-algebras close to them.

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Alberto Elduque

University of Zaragoza, Spain

Gradings on composition algebras

The group gradings on symmetric composition algebras will be classified. Applications of this result, and of the gradings on octonions, to gradings on the exceptional simple Lie algebras will be considered.

T. Foth[1] and M. Tvalavadze[2]

[1] *University of Western Ontario; University of Saskatchewan*

On varieties parameterizing graded complex Lie algebras

The variety of Lie algebra structures on a finite-dimensional complex vector space has been studied extensively, see, in particular, [1, 2, 5, 6, 8]. This is closely related to deformation theory of Lie algebras, going back to foundational works [4, 7].

In this note we study certain subvarieties of the varieties studied by Kirillov, Neretin et al. They appear as varieties parameterizing contractions [3] of G-graded finite-dimensional complex Lie algebras (here G is a finite abelian group). To explain informally, a finite-dimensional complex vector space and a G-grading are fixed, and structure constants are allowed to vary.

Let G be a finite abelian group, and R a fixed G-graded finite-dimensional complex Lie algebra. Denote by X_R the complex projective variety that parameterizes (up to rescaling all structure constants simultaneously by a non-zero complex number) contractions of R in the sense of [3]. Let $R^{(\varepsilon)}$ be a contraction of R corresponding to a parameter $\varepsilon \in X_R$. In this paper we study X_R (a subvariety of $\mathbb{P}^{n^2-1}$) and $R^{(\varepsilon)}$ from geometric as well as algebraic point of view.

References

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Plamen Koshlukov

University of Campinas, Brazil

Identities in graded tensor products of PI algebras

Let K be a field of characteristic 0. We study the polynomial identities satisfied by $\mathbb{Z}_2$-graded tensor products of T-prime algebras. Regev and Seeman proved that in a series of cases such tensor products are PI equivalent to T-prime algebras; they conjectured that this is always the case. We deal here with the remaining cases and thus confirm Regev and Seeman's conjecture. For some "small" algebras we can remove the restriction on the characteristic of the base field, and we show that the behaviour of the corresponding graded tensor products is quite similar to that for the usual (ungraded) tensor products. More precisely we prove that if the base field is infinite and of characteristic $p > 2$ then the T-ideal of the algebra $M_2(E)$ is strictly contained in the T-ideal of $M_{1,1}(E)\bar{\otimes}E$ where E is the infinite-dimensional Grassmann algebra, and $\bar{\otimes}$ is the graded tensor product. Recall that in characteristic 0 the above two algebras are PI equivalent also when one considers the ordinary tensor product but this is not the case in positive characteristic.

Finally we consider β-graded tensor products (also called commutation factors) and their identities. We show that Regev's $A \otimes B$ Theorem holds for β-graded tensor products whenever the gradings are by finite abelian groups. Furthermore we study the PI equivalence of β-graded tensor products of T-prime algebras.

M. Kotchetov

Memorial University of Newfoundland

Formal group actions and gradings by p-groups in characteristic p

We are interested in describing all group gradings on finite-dimensional simple Lie algebras over an algebraically closed field F. In the case $\operatorname{char} F = 0$, all gradings on the classical simple Lie algebras, except of type D_4, have been described in the works of J. Patera, H. Zassenhaus, M. Havlíček, E. Pelantová (fine gradings, i.e., the gradings that cannot be refined) and Yu. Bahturin, M. Zaicev, I. Shestakov (all gradings). A description of fine gradings on the Lie algebra of type D_4 has recently been announced by C. Draper and M. Cándido.

It turns out that the description given by Yu. Bahturin, M. Zaicev and I. Shestakov is also valid in the case $\operatorname{char} F = p > 0$, $p \neq 2$. Gradings by groups whose order is not divisible by p can be treated in the same way as in the case $\operatorname{char} F = 0$. Our approach to gradings by p-groups is based on interpreting them in terms of actions of formal groups or, more precisely, of the connected cocommutative Hopf algebras associated to them (so-called hyperalgebras). The main ingredient of the proof is to show that, under certain conditions, any such action can be lifted from a Lie subalgebra $L \subset M_n(F)$ to the entire matrix algebra $M_n(F)$. This result applies to the types A_n, B_n, C_n, D_n except A_n for $n = pk - 1$. In this latter case the algebra $sl_{n+1}(F)$ contains the scalar matrices, so we consider the quotient $psl_{n+1}(F)$ instead. The results in this talk are joint work with Yu. Bahturin and S. Montgomery.

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Dimitry Leites

University of Stockholm, Sweden

Towards classification of simple finite dimensional modular Lie superalgebras

A way to construct (conjecturally all) simple finite dimensional modular Lie (super)algebras over algebraically closed fields of characteristic not 2 is offered.

In characteristic 2, the method is supposed to give only simple Lie (super)algebras graded by integers and only some of the non-graded ones). The conjecture is backed up with the latest results computationally most difficult of which are obtained with the help of Grozman's software package SuperLie.

Certain phenomena pertaining to characteristic 2 is unexpected: for example, the adjoint representation of a simple Lie superalgebra may have non-trivial submodules. This, of course, affects some of conventional constructions.

But most interesting and basic question which seems to me open is:

WHAT IS THE "CORRECT" DEFINITION OF THE UNIVERSAL ENVELOPING ALGEBRA IN POSITIVE CHARACTERISTIC?

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Jason McGraw

Memorial University

Gradings of Lie algebras of Cartan type

In this talk we will discuss gradings by finite abelian groups on the Witt algebras where the characteristic of the field, p, is greater than 3. First we describe all gradings of $W(m;1)$ by finite abelian groups whose order isn't divisible by p. This is accomplished by finding the conjugacy classes of the automorphisms of $W(m;1)$. We then proceed to describe all gradings of $W(1;1)$ by finite abelian groups whose order is divisible by p using semisimple derivations of $W(1;1)$. We will show how to construct similar gradings of $W(m;1)$ by groups of finite order divisible by p similar to those of the $W(1;1)$ case. Finally we will discuss how to generalize these techniques used in this talk to find gradings of $W(m;\underline{n})$. Knowing the gradings of $W(m;\underline{n})$ would provide insight for finding the gradings of the other Lie algebras of Cartan type.

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Vincenzo Nardozza

University of Bari, Italy

Graded algebras and tensor product theorems

Let F be a field of characteristic zero and let E be the Grassmann algebra of an infinite-dimensional vector space. The non trivial verbally prime algebras $M_n(F)$, $M_n(E)$ and $M_{p,q}(E) \subseteq M_n(E)$ (for $n = p + q$) possess a natural superalgebra structure. By suitably combining an elementary $\mathbb{Z}_n$-grading with the $\mathbb{Z}_2$-grading one gets a *near-elementary* grading by the group $G := \mathbb{Z}_n \times \mathbb{Z}_2$. These gradings can be defined also for both the tensor products and super-tensor products of verbally prime algebras, and their graded polynomial identities can be determined. This approach has a twofold consequence about ordinary polynomial identities:

(1) it allows a direct proof of Kemer's Tensor Product Theorem, and of the so called Kemer's PI-equivalences among verbally prime algebras and their tensor products;

(2) it provides an answer to a conjecture raised by Regev and Seeman about the super-tensor product of verbally prime algebras. It turns out that the super-tensor product $A\hat{\otimes}B$ of the verbally prime algebras A, B is indeed PI-equivalent to a verbally prime algebra. Actually, the PI-equivalences among verbally prime algebras and their super-tensor products are explicitly determined.

This method applies in case F is infinite of characteristic different from 2, considering the multi-linear polynomial identities of the algebras, that is comparing algebras through the so called *multi-linear equivalence*.

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Michael Natapov
(joint work with Eli Aljadeff and Darrell Haile)

Indiana University - Bloomington, USA

On graded polynomial identities of matrices.

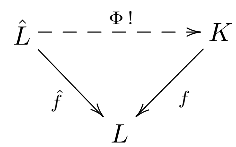
Let G be a group and $A = M_n(F)$ be a full $n \times n$ matrix algebra over an algebraically closed field F. A G–grading on A is called fine if $\dim_F(A_g) \leq 1$. It is not difficult to see that a G–grading on A is fine if and only if the support of the grading $H = \{g \in G \mid A_g \neq 0\}$ is a subgroup of G and A is isomorphic to the twisted group algebra F^cH for an appropriate two-cocycle $c \in Z^2(H, F^\times)$. We refer to such group H as a group of central type. Given a fine grading on A supported by a group of central type H, let $F\langle x_{i,g} | \; g \in H, i = 1, 2, \ldots\rangle$ be an associative H–graded free algebra, and T_H be the T–Ideal of H–graded polynomial identities of A. Unlike the classical (non-graded) case the relatively free algebra $F\langle x_{i,g}\rangle/T_H$ may or may not be a domain. It is a domain exactly when H is on a precise list of families of nilpotent groups, called Λ. We describe generating set for T_H where H is any group of central type, and give minimal generating sets for groups on the list Λ (joint with Haile).

Erhard Neher

University of Ottawa

Universal central extensions of graded Lie algebras

A *central extension* of a Lie algebra L is a surjective Lie algebra homomorphism $f : K \to L$ whose kernel lies in the centre of K. A *universal central extension* is the biggest central extension in the following sense: A central extension $\hat{f} : \hat{L} \to L$ such that for every other central extension $f : K \to L$ there exists a unique Lie algebra homomorphism $\Phi : \hat{L} \to K$ satisfying $\hat{f} = f \circ \Phi$.



Such a universal extension exists iff L is perfect, and in this case it is unique up to isomorphism. If L is graded by some abelian group then so is its universal central extension (if it exists).

To consider a central extension of a Lie algebra, is one of the standard techniques in the theory of infinite-dimensional Lie algebras (it is not important for finite-dimensional semisimple Lie algebras since they coincide with their universal central extension). One of the reasons why universal central extensions are so important is that they often behave much better than the original Lie algebra, e.g. their representation theory is more tractable. A case in point is the class of affine Lie algebras, for which universal central extensions are a basic ingredient in their construction. For example, in the untwisted case, one starts with a simple complex finite-dimensional Lie algebra $\mathfrak{g}$, forms the loop algebra $L = \mathfrak{g} \otimes_{\mathbb{C}} \mathbb{C}[t^{\pm 1}]$, and then constructs the universal central extension $\hat{L}$ of the complex Lie algebra L, which turns out to be $\hat{L} = L \oplus \mathbb{C}c$ on the vector space level (the algebra product is a little bit more complicated). To finish the construction, the untwisted affine Lie algebra is then the semidirect product $\hat{L} \oplus \mathbb{C}d$, where d acts on L as a degree derivation and kills c.

Problem: How can one construct a central extension, or even a universal central extension of a concrete Lie algebra?

There are several solutions of this problem. In the talk, the speaker's most favorite constructions of (universal) central extensions of graded Lie algebras will be described. They use so-called 2-cocycles and derivations, and provide a rather concrete working model for the universal central extensions of certain graded Lie algebras of present interest.

One of them is the class of extended affine Lie algebras which are a natural "higher nullity" analogue of affine Lie algebras. They are constructed in a way similar to the affine Lie algebra: One considers the universal central extension

$\hat{L}$ of a so-called Lie torus L (basic example: $L = \mathfrak{g} \otimes_{\mathbb{C}} \mathbb{C}[t_1^{\pm 1}, t_2^{\pm 1}, \ldots, t_n^{\pm 1}]$ for $\mathfrak{g}$ as above) and adds special "degree" derivations to $\hat{L}$.

Time permitting, we will also describe the universal central extension of the so-called multiloop Lie algebras, closely related to, but different from Lie tori. These are certain subalgebras of $L = \mathfrak{g} \otimes_{\mathbb{C}} \mathbb{C}[t_1^{\pm 1}, t_2^{\pm 1}, \ldots, t_n^{\pm 1}]$ for $\mathfrak{g}$ as above, defined in terms of a grading of $\mathfrak{g}$ by a finite abelian group.

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Filtrations in semisimple rings and Lie algebras

These results are, for the most part, joint with Yiftach Barnea and are several years old. Still, there is at least one new fact to report. Let $\mathbb{Z}$ be the ring of integers and let A be an algebra, associative or Lie, over a field K. A $\mathbb{Z}$-filtration $\mathcal{F}$ of A is a collection $\mathcal{F} = \{F_i \mid i \in \mathbb{Z}\}$ of subspaces such that $F_i \subseteq F_{i+1}$ and $F_i{\cdot}F_j \subseteq F_{i+j}$. We say that $\mathcal{F}$ is bounded if there exist subscripts ℓ, ℓ' with $F_\ell = 0$ and $F_{\ell'} = A$. A second filtration $\mathcal{G} = \{G_i \mid i \in \mathbb{Z}\}$ is said to be larger than $\mathcal{F}$ if $G_i \supseteq F_i$ for all i. The goal of this work is to classify all maximal bounded filtrations when A is a semisimple associative algebra or a semisimple Lie algebra over the complex numbers. This problem was motivated by a result of Barnea, Shalev and Zelmanov on maximal graded subalgebras of Kac-Moody Lie algebras, and it essentially completes their classification.

While the associative results are fairly routine and expected, the Lie results seem to tap a new vein in the study of root systems of simple Lie algebras. Indeed in the latter case, with a few possible exceptions, we have the following result, where we let V denote the real vector space spanned by the roots ϕ of A.
Theorem. Let A be a simple finite-dimensional Lie algebra over the complex numbers. If $\mathcal{F}$ is a maximal bounded filtration of A, then: (i) $F_0 \supseteq L_0$, a Cartan subalgebra of A; (ii) there exists a linear functional $\lambda\colon V \to \mathbb{R}$ so that $F_i = \sum_{\lambda(\phi)\le i} L_\phi$, a sum of suitable root spaces; (iii) the functionals that occur in this manner are precisely the "rigid" ones, namely those that take on integer values on a basis of V consisting of roots; (iv) such rigid functionals can be described in terms of the maximal roots of A; and (v) there is a semisimple subalgebra $P_\lambda = \sum_{\lambda(\phi)\in\mathbb{Z}} L_\phi$ of A of maximal rank, associated to λ, and these are precisely the "one-step" subalgebras in Dynkin's description.

Ivan Shestakov

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The Coordinatization Theorem for Jordan Algebras and Superalgebras

The Jacobson Coordinatization Theorem is one of corner stone in the theory of Jordan algebras and their representations. In particular, it establishes a Morita-equivalence between the categories of Jordan modules over a Jordan matrix algebra and of alternative bimodule with nuclear involution over the corresponding coordinatizating alternative algebra with involution. We will speak on a generalization of this theorem for Jordan superalgebras and on applications to representations of Jordan matrix superalgebras.

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The skew-symmetric identities of octonions

Quadratic alternative superalgebras will be considered and their super - identities and central functions on one odd generator will be described. As a corollary, all multilinear skew-symmetric identities and central polynomials of octonions are classified.

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Locally affine root systems and locally affine Lie algebras

There is a correspondence between locally finite irreducible root systems (as a natural generalization of classical finite irreducible root systems) and locally finite split simple Lie algebras (as a natural generalization of finite-dimensional split simple Lie algebras) described by Neeb and Stumme. Also, there is a correspondence between affine root systems (in the sense of Macdonald) and affine Kac-Moody Lie algebras (or loop algebras). I will explain about a natural generalization of these two correspondences, namely, a correspondence between "locally affine root systems" and "locally affine Lie algebras" (or "locally loop algebras").

Summary Schedule

	Friday 08/29 AA-1043	**Saturday 08/30 AA-1043**	**Monday 09/01 AA-1043**	**Tuesday 09/02 C-2045**
9:00 - 9:30	**Opening**	*Coffee*	*Coffee*	*Coffee*
9:30 - 10:20	Benkart	Shestakov	Benkart	Shestakov
10:20 - 10:30	*Break*	*Break*	*Break*	*Break*
10:30 - 11:20	Leites	Drensky	Leites	Drensky
11:20 - 11:40	*Coffee*	*Coffee*	*Coffee*	*Coffee*
11:40 - 12:30	Elduque	Passman	Smirnov	Koshlukov
12:30 - 2:30	*Lunch*	*Lunch*	*Lunch*	*Lunch*
2:30 - 3:20	Neher	Aljadeff	Bresar	Kotchetov
3:20 - 3:30	*Break*	*Break*	*Break*	*Break*
3:30 - 4:10	Yoshii	Nardozza	Caveney	McGraw
4:10 - 4:30	*Coffee*	*Coffee*	*Coffee*	**Closure**
4:30 - 5:10	Draper	Natapov	Tvalavadze	