

# FINITE FREE RESOLUTIONS

JUERGEN HERZOG

## LECTURE 1: ACYCLICITY CRITERIA FOR FINITE FREE RESOLUTIONS AND THE LOWER BOUND CONJECTURE FOR BETTI-NUMBERS

We discuss the acyclicity criteria of Peskine-Szpiro and Buchsbaum-Eisenbud and consider the special case of the Hilbert-Burch theorem, and then describe what is known about the challenging Buchsbaum-Eisenbud and Horrocks conjecture regarding lower bounds for the Betti numbers of a finite free resolution.

## LECTURE 2: BOIJ-SÖDERBERG THEORY WITH APPLICATION TO THE MULTIPLICITY THEOREM

We consider graded free resolutions of graded modules over the polynomial ring, especially pure resolutions which are the building blocks of Boij-Söderberg theory. We derive the Herzog-Kühl formulas and the Huneke-Miller formula the multiplicity of ideals with pure resolution, and show how Boij-Söderberg theory is used to prove the Huneke-Srinivasan multiplicity conjecture.

## LECTURE 3: GENERIC INITIAL IDEALS OF GRADED IDEALS AND BETTI NUMBERS OF IDEALS WITH LINEAR RESOLUTION

We recall the Bayer-Charalambous-Popescu theorem about extremal Betti numbers and characterize the graded ideals with the property that all its graded Betti numbers are stable when passing to the generic initial ideal, and also present an extension of this result as it is given by Conca-Herzog-Hibi. Finally following a recent paper of Varbaro, Sharifan and myself we characterize all possible Betti numbers of ideals with linear resolution.

LECTURE 4: MONOMIAL IDEALS WITH THE PROPERTY THAT ALL ITS MONOMIAL  
LOCALIZATIONS HAVE A LINEAR RESOLUTION

In this lecture a conjecture by myself and Bandari will be presented. The conjecture states that a monomial ideal and all its monomial localizations have linear resolution if and only if the ideal is polymatroidal. Known cases, including the case of squarefree monomial ideals, are presented and related questions and conjectures are discussed.

FACHBEREICH 6, MATHEMATIK UNIVERSITÄT DUISBURG-ESSEN, CAMPUS ESSEN 45117 ESSEN,  
GERMANY

*E-mail address:* `juergen.herzog@uni-essen.de`