GENERIC INITIAL IDEALS

GIULIO CAVIGLIA

In a series of three lectures I will give an overview of the notion of generic initial ideal and discuss some applications to the problem of bounding invariants derived from graded free resolutions.

In the first lecture I will present some basic properties of generic initial ideals; for instance, they are fixed under the action of the Borel subgroup of the general linear group, as shown by Galligo. Because of this fact, they have particularly nice combinatorial properties which, unfortunately, depend on the characteristic of the base field. I will prove a theorem of Bayer and Stillman, which states that taking the generic initial ideal, with respect to the reverse lexicographic order, preserves important homological invariants such as the Castelnuovo-Mumford regularity and the projective dimension. Furthermore, I will discuss a result of Bayer, Charalambous, and Popescu, which shows that also extremal Betti numbers are preserved when taking the generic initial ideal with respect to the reverse lexicographic order.

In the second lecture I will show how to use generic initial ideals to derive, in any characteristic, a double exponential bound for the Castelnuovo-Mumford regularity of homogeneous ideals in terms of their generating degrees; I will recover in this way results of Giusti-Galligo, Bayer-Mumford, and Sbarra and myself.

In the third lecture I will consider the problem of finding upper bounds for Betti numbers. I will use the techniques developed in the first two lectures to give a self-contained proof, in characteristic zero, of a theorem of Mermin and Murai. Their result, which is a special case of the Lex-Plus-Power conjecture, provides a sharp upper bound for the Betti numbers of ideals containing the pure powers of the variables in terms of their Hilbert functions.

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