
POLYNOMIAL IDENTITIES IN ALGEBRAS

POLYNOMIAL IDENTITIES IN ALGEBRAS
Abstracts of International Workshop at Memorial
University of Newfoundland, August
29, 2002 - September 3, 2002

Edited by
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Preface

Dear guests! It is a pleasure to see you all in St. John's, the capital of Canadian province of Newfoundland and Labrador. The International Workshop "Polynomial Identities in Algebras" is hosted by the Department of Mathematics and Statistics of Memorial University of Newfoundland, the east most University of North America. In this package you will find the the schedule of talks and their abstracts, the list of participants and other useful materials.

THE EDITOR

SCHEDULE OF THE WORKSHOP

AUGUST 29, 2002

- 9:30 - 10:30** ANTONIO GIAMBRUNO: *Standard Polynomials, Capelli Polynomials And Asymptotics.*
COFFEE BREAK
- 11:00 - 12:00** AMITAI REGEV: *A_n -Codimensions And Cocharacters.I*
LUNCH BREAK
- 2:15 - 3:15** VESSELIN DRENSKY: *Behavior Of Multiplicities In Cocharacters Of PI-algebras. I*
-
- 3:30 - 4:00** JENO SZIGETI: *Computing With Matrices Over The Grassmann Algebra.*
COFFEE BREAK
- 4:15 - 4:45** ALAIN D'AMOUR: **-Polynomial Identities Of Matrices: The Low Degrees.*
- 4:50 - 5:20** HEYDAR RADJAVI: *Polynomial Identities And Reducibility of Semigroups.*

WINE AND CHEESE PARTY: 6 - 8 P.M. AT BAHTURINS'

AUGUST 30, 2002

9:30 - 10:30 ALEXANDER KEMER: *Conjecture of Procesi*
COFFEE BREAK

11:00 - 12:00 ED FORMANEK: *The ring of generic matrices.I*
LUNCH BREAK

2:15 - 3:15 VIKTOR LATYSHEV: *Specht's Problem
in Positive Characteristic*

3:30 - 4:00 ADALBERT BOVDI: *Applications of the group
identities theory to the group
of units of group algebras.*
COFFEE BREAK

4:15 - 4:45 ANGELA VALENTI: *Gradings and graded identities
of the algebra of $n \times n$ upper
triangular matrices.*

4:50 - 5:20 ROLF REES: *A polynomial problem arising
from t -designs.*

5:25 - 5:55 ROBERTO LA SCALA : *Knuth-Robinson-Schensted
correspondence and Weak Polynomial
Identities of $M_{1,1}(E)$*

WORKSHOP DINNER: 6:30 P.M. IN THE FACULTY CLUB

AUGUST 31, 2002

DAY TOUR TO SEABIRD SANCTUARY AT CAPE ST. MARY.
START AT 10:00 FROM PRESCOTT INN ON MILITARY ROAD.

SEPTEMBER 1, 2002

- 9:30 - 10:30** MIKHAIL ZAICEV: *Numerical characteristics and extremal varieties. I.*
COFFEE BREAK
- 11:00 - 12:00** AMITAI REGEV: *A_n -codimensions and cocharacters.II*
LUNCH BREAK
- 2:15 - 3:15** VESSELIN DRENSKY: *Combinatorics of Words and Constructions in Ring Theory.II.*
-
- 3:30 - 4:00** VIKTOR BOVDI: *Symmetric units and group identities*
COFFEE BREAK
- 4:15 - 4:45** LEONID KROP: *Quantum Hyperalgebra of Rank 1.*
- 4:50 - 5:20** MURRAY BREMNER: *Quantization of Lie and Jordan triple systems.*

SEPTEMBER 2, 2002

- 9:30 - 10:30** YURI BAHTURIN: *Generalized Lie Solvability of Associative Algebras. I.*
COFFEE BREAK
- 11:00 - 12:00** ED FORMANEK: *The Ring Of Generic Matrices.II*

CITY AND NEIGHBORHOOD TOURS IN THE AFTERNOON.

WINE AND CHEESE PARTY: 6 - 8 P.M. AT GASKILLS.

SEPTEMBER 3, 2002

- 9:30 - 10:30** MIKHAIL ZAICEV: *Numerical characteristics and extremal varieties. II.*
COFFEE BREAK
- 11:00 - 12:00** SERGEI MISHCHENKO: *Almost Polynomial Growth Varieties of Linear Algebras.*
LUNCH BREAK
- 2:15 - 3:15** YURI BAHTURIN: *Generalized Lie Solvability of Associative Algebras. II.*
-

- 3:30 - 4:00** MIKHAIL KOTCHETOV: *Identities of the smash product of a group algebra and the universal envelope of a Lie superalgebra.*
COFFEE BREAK
- 4:15 - 4:45** VINCENZO NARDOZZA: *Proper subvarieties of \mathfrak{N}_5 .*
- 4:50 - 5:20** PLAMEN KOSHLUKOV: *Identities in certain algebras in positive characteristic.*

CLOSING OF THE WORKSHOP

GENERALIZED LIE SOLVABILITY OF ASSOCIATIVE ALGEBRAS

Yuri Bahturin

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These lectures will be devoted to discussing some new methods and approaches to polynomial identities in associative algebras arising from recent studies on bicharacters and twisting in algebras with actions and coactions of Hopf algebras, in the spirit of the works of M. Scheunert (Bahturin - Fischman - Montgomery, Etingof - Zhelaki, etc.), and the classification results about group gradings of matrix algebras (Bahturin - Sehgal - Zaicev). Combining these results, in two joint papers, one with M. Parmenter and the other with S. Montgomery, and M. Zaicev, we can give answers to the questions about the generalized Lie structure of certain associative algebras. Given a skew-symmetric bicharacter $\beta : H \times H \rightarrow k^*$ of a Hopf algebra H over a field k and an associative algebra A in the category \mathcal{M}^H of right H -comodule algebras one can define a new operation, called the β -bracket, as follows:

$$[a, b]_\beta = ab - \sum_{a,b} \beta(a_{(1)}, b_{(1)}) b_{(0)} a_{(0)}$$

where we use standard notation for the comodule map $\rho : A \rightarrow A \otimes H$:

$$\rho(a) = \sum_a a_{(0)} \otimes a_{(1)} \text{ where } a, a_{(0)} \in A, a_{(1)} \in H.$$

In the case where H is a cotriangular Hopf algebra we have that $(A, [,]_\beta)$ is a generalized Lie algebras in the sense that it satisfies generalized anti-commutativity (β -anti-commutativity)

$$[a, b]_\beta = - \sum_{a,b} \beta(a_{(1)}, b_{(1)}) [b_{(0)}, a_{(0)}]_\beta$$

and β -Jacobi identity

$$\begin{aligned} \beta(c_{(1)}, a_{(1)})[[a_{(0)}, b]_{\beta}, c_{(0)}]_{\beta} &+ \beta(a_{(1)}, b_{(1)})[[b_{(0)}, c]_{\beta}, a_{(0)}]_{\beta} \\ &+ \beta(b_{(1)}, c_{(1)})[[c_{(0)}, a]_{\beta}, b_{(0)}]_{\beta} = 0. \end{aligned}$$

In this lecture we consider generalized Lie algebra structures on graded associative algebras, that is when $H = k[G]$, G an abelian group. We are interested in the situation where such structures are solvable and even commutative. We first formulate some theorems in the case of solvable structures on semi prime, prime or simple associative algebras. Then we describe finite-dimensional associative algebras over an algebraically closed field of characteristic zero graded by a finite elementary abelian group which are generalized commutative under a skew-symmetric bicharacter on the grading group. A special case is where A is a group algebra (of some other group) itself and we give some information about this case, as well.

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- [2] Bahturin, Y., Mikhalev, A., Petrogradsky, V., Zaicev, M., *Infinite-Dimensional Lie Superalgebras*. Walter De Gruyter Expositions in Math., **9** (1992).
- [3] Bahturin, Y., Montgomery, S., *PI-envelopes of Lie superalgebras*, Proc. AMS, **127** (1999), 2829 - 2839.
- [4] Bahturin, Y., Montgomery, S., Zaicev M., *Generalized Lie Solvability of Associative Algebras*, In: "Groups, Rings, Lie and Hopf Algebras". Proc. Int. Workshop held in St. John's, Maay 28 - June 1, 2001, to appear.
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- [7] Bahturin Y., Sehgal S., Zaicev M. *Group gradings on associative algebras*, J. Algebra, **241**(2001), 677 - 698.
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APPLICATIONS OF THE GROUP IDENTITIES THEORY TO THE GROUP OF UNITS OF GROUP ALGEBRAS

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Hungary

The interest for modular group algebra theory has appeared in 70-th due to results of the D.Passman and some Lie properties of the group algebras were described. In that time we find close connection between Lie properties of the group algebra and its group of units, and jointly with I.Khripta we described the group algebra of torsion groups with solvable group of units. The Lie methods played a decisive role in the investigation group of units, as shows the research done by A.Shalev. The theory group of units has undergone significant changes in the past 10 year. The new progress in the theory of group identities, devoted by several authors, gives new results in the theory of group of units.

Using the theory of the group identities, we have obtained complete description of the group algebras of arbitrary groups with either solvable or n -Engel condition for the group of units. For example:

Let G be a group with infinite Sylow p -subgroup and F be the field of characteristic p . The group of units $U(FG)$ is n -Engel if and only if the algebra FG is n -Engel i.e. G is a nilpotent group and G has a normal subgroup A such that the commutator subgroup A' is a finite p -group.

SYMMETRIC UNITS AND GROUP IDENTITIES

Victor Bovdi

University of Debrecen

Hungary

Let $U(R)$ be the group of units of a ring R with involution $*$ and $U^+(R) = \{u \in U(R) \mid u = u^*\}$ be the set of the symmetric units of R . A natural question is to characterize those rings R such that the set $U^+(R)$ of the symmetric units satisfies a group identity. In our talk we discuss the properties of the group algebras FG of the groups G over the field F of characteristic p , whose symmetric units satisfies a group identity. In particular, we obtain a generalization of results Giambruno-Sehgal-Valenti (see [1]) in case when FG is a either modular or semiprime group algebra of a non-torsion group G over a field F of characteristic p .

References

- [1] Giambruno, A.; Sehgal, S. K.; Valenti, A., *Symmetric units and group identities*, Manuscripta Math. **96**(1998), 443–461.

QUANTIZATION OF LIE AND JORDAN TRIPLE SYSTEMS

Murray Bremner

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This talk will show how the decomposition of the group ring of the symmetric group into a direct sum of full matrix subrings can be used to give a complete classification of n -ary operations. Roughly speaking, row equivalence of matrices corresponds to quasi-equivalence of operations. In particular, the Lie and Jordan products represent the two non-trivial quasi-equivalence classes of binary operations. For ternary operations, there are infinitely many quasi-equivalence classes, which divide into eight classes, and four infinite families of classes each with a single parameter. The Lie triple product is contained in one of the infinite classes, and the other operations in the class can be regarded as quantizations of that product. Similar remarks apply to the Jordan triple product. For special values of the parameter, the operation satisfies an identity of degree 5. This identifies some new ternary operations which define varieties of triple systems, similar to Lie and Jordan triple systems, which seem to be an interesting direction for further research.

THE RING OF GENERIC MATRICES

Edward Formanek

PennState University

The ring of $n \times n$ generic matrices and the generic division ring were first named by Procesi in 1966, but they had already been studied by Amitsur in 1955 as a relatively free ring and its Ore ring of fractions. They play an important role in the theory of PI-rings, the study of finite-dimensional division algebras, and the invariant theory of $n \times n$ matrices. My first talk will be a historical survey. Although most of what is known about the ring of generic matrices was established in the last fifty years, work of Sylvester, Fricke and Klein on 2×2 matrices in the nineteenth century anticipated some later developments. In the twentieth century, there were early relevant works by Dubnov in Russia and a few physicists in the in the United States. My second talk will be about what I consider the most significant open question about generic matrices: *Is the center of the $n \times n$ generic division ring a rational function field?* An answer, positive or negative, would have important consequences. A positive answer would imply the Mercuriev-Suslin theorem that the Brauer group is generated by cyclics in the presence of enough roots of unity. A negative answer would give naturally occurring examples of unirational fields which are not rational. Progress on this question has been slow.

STANDARD POLYNOMIALS, CAPELLI POLYNOMIALS AND ASYMPTOTICS

Antonio Giambruno

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Let $M_k(F)$ be the algebra of $k \times k$ matrices over a field F of characteristic zero. It is well known that $M_k(F)$ satisfies St_{2k} , the standard polynomial of degree $2k$, and Cap_{k^2+1} , the $(k^2 + 1)$ -Capelli polynomial. For a polynomial f , let $\langle f \rangle_T$ be the T-ideal of the free algebra generated by f . Even if St_{2k} and Cap_{k^2+1} do not generate $Id(M_k(F))$, the T-ideal of identities of $M_k(F)$, there is a general feeling that $\langle St_{2k} \rangle_T$ and $\langle Cap_{k^2+1} \rangle_T$ should be quite close to $Id(M_k(F))$.

We study the asymptotic behavior of the sequence of codimensions of the T-ideal generated by a standard or a Capelli polynomial and we compare it with the corresponding asymptotics of $Id(M_k(F))$. In this study two more algebras come into play: the algebra of $(k + l) \times (k + l)$ matrices having the last l rows and the last k columns equal to zero and the algebras of upper block-triangular matrices.

CONJECTURE OF PROCESI

Alexander Kemer
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Let R_Z, R_F are k generated algebras of generic matrices of order n over the ring of integers Z and over an infinite field F of characteristic p respectively. The conjecture of C. Procesi [1] is well-known: A kernel of the canonical epimorphism $R_Z \rightarrow R_F$ is equal to pR_Z .

Let k, p, n are given. Denote by ψ the natural homomorphism

$$Z\langle x_1, \dots, x_k \rangle \rightarrow F\langle x_1, \dots, x_k \rangle.$$

The conjecture of C. Procesi can be also formulated in the following form: If the algebra $M_n(F)$ satisfies identity $\psi(f) = 0$ then $f = pg$ modulo $T[M_n(Q)]$ for some polynomial $g \in Z\langle x_1, \dots, x_k \rangle$.

In [2] C. Procesi has proved his conjecture for $n = 2, p > 2$ and every k . In 1985 W. Schelter [3] gave a negative answer to this conjecture for $p = 2, n = 2$.

Recently the author gave a negative answer in every characteristic.

Theorem 1 *For every prime p there exist $n \leq p$ and multilinear polynomial $f \in Z\langle X \rangle$ such that the algebra $M_n(F)$ satisfies identity $\psi(f) = 0$ but $f \neq pg$ modulo $T[M_n(Q)]$ for every polynomial $g \in Z\langle X \rangle$.*

We also proved some positive results in the case $n = 3$.

Theorem 2 *For $n = 3, k = 2$ and for every sufficiently large prime p the conjecture of Procesi has a positive answer.*

Theorem 3 *For every k and for every sufficiently large prime p the following statement is true: If the algebra $M_3(F)$ satisfies identity $\psi(f) = 0$ then there exist some polynomial g with integer coefficients such that $M_3(Z)$ satisfies a weak identity $f = pg$.*

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IDENTITIES OF THE SMASH PRODUCT OF A GROUP ALGEBRA AND THE UNIVERSAL ENVELOPE OF A LIE SUPERALGEBRA

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Suppose H is a Hopf algebra, A an H -module algebra. Then we can form the smash product $A\#H$, which can be viewed as a “deformation” of the usual tensor product $A \otimes H$ (the latter arises when the action of H on A is trivial). In particular, if H is the group algebra of a group G that acts on A by automorphisms, then $A\#H$ is the so called skew group ring of G with coefficients in A .

We study the question when $A\#H$ satisfies a polynomial identity. An obvious necessary condition is that both A and H must be *PI*. In the case of tensor product, this is also sufficient (by a theorem of A.Regev). In general, we have to take into account the structure of the action of H on A . To this end, we define an appropriate kind of delta-sets and obtain some necessary conditions on the action of H on A in terms of these delta-sets for a certain class of algebras.

Then we specialize to the case when H is a group algebra, i.e. $A\#H$ is a skew group ring. The existence of a polynomial identity for such $A\#H$ in the case of semiprime A was considered by D.Passman. Recently, Yu.Bahturin and V.Petrogradsky gave explicit necessary and sufficient conditions for $A\#H$ to be *PI*, where A is the restricted envelope of a p -Lie algebra (hence not semiprime in general). We apply our results on delta-sets, together with known facts about group algebras and universal envelopes, to find explicit necessary and sufficient conditions for $A\#H$ to be *PI*, where A is the universal envelope of a Lie superalgebra.

IDENTITIES IN CERTAIN ALGEBRAS IN POSITIVE CHARACTERISTIC

Plamen Koshlukov

We discuss identities in associative algebras over infinite fields of characteristic $p \neq 2$. First, using descriptions of the 2-graded identities satisfied by the algebras $M_{1,1}(E)$ and $E \otimes E$, we show that their T-ideals do not coincide if $p \neq 0$. Here E is the unitary Grassmann algebra of an infinite dimensional vector space. If one considers nonunitary Grassmann algebras, the above two algebras share the same T-ideal as it is the case in characteristic 0. Furthermore we study graded identities of the algebras $M_{1,1}(E) \otimes E$ and $M_2(E)$, and show that these satisfy different graded identities if one considers certain natural gradings by the cyclic group of order 4. This fact holds in positive characteristic only.

QUANTUM HYPERALGEBRAS OF RANK 1

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We study the algebra U obtained via Lusztig's "integral" form of the generic quantum algebra for the Lie algebra $g = sl_2$ modulo the two-sided ideal generated by $K^l - 1$. We show that U is a smash product of the quantum deformation of the restricted universal enveloping algebra u of g modulo $K^l - 1$ and the ordinary universal enveloping algebra \bar{U} of g . We show that the above smash product reduces to the tensor product modulo the nilpotent radical of u , and we compute the primitive (= prime) ideals of U . Next we describe a decomposition of u into the simple \bar{U} -submodules.

For a primitive central idempotent ϵ of u we show that ϵU is an indecomposable two sided ideal of U . The center of U is explicitly computed.

Some sublattices of ideals of U are classified, notably the sublattice of cofinite ideals.

KNUTH-ROBINSON-SCHENSTED CORRESPONDENCE AND WEAK POLYNOMIAL IDENTITIES OF $M_{1,1}(E)$

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In this talk we show that the ideal I_w of the weak polynomial identities of the superalgebra $M_{1,1}(E)$ is generated by the proper polynomials $[x_1, x_2, x_3]$ and $[x_2, x_1][x_3, x_1][x_4, x_1]$. This is proved for any infinite field F of characteristic different from 2. Precisely, if B is the subalgebra of the proper polynomials of the free algebra $F\langle X \rangle$, we determine a basis and the dimension of any multihomogeneous component of the quotient algebra $B(I_w) = B/B \cap I_w$. We compute also the Hilbert series of this algebra. The arguments we use for proving such results are essentially the following. The first step consists in proving that, by means of the identities $[x_1, x_2, x_3]$ and $[x_2, x_1][x_3, x_1][x_4, x_1]$, a generating set for $B(I_w)$ is given by products of commutators of length 2 which satisfy some combinatorial conditions. We call such “standard products” *c-arrays*. We develop a variant of the Knuth-Robinson-Schensted algorithms that establishes a correspondence between *c-arrays* and semistandard tableaux of double shape. We prove then that the multilinear components of the algebra $B(I_w)$ are isomorphic to the multilinear components of the invariant algebra of the orthogonal group $O(2, F)$ with respect to a suitable (multi)graduation. For such algebra and $\text{char}(F) \neq 2$, De Concini & Procesi proved in fact that there exists a “standard basis” parametrized by semistandard tableaux of double shape (the standardity is here according to the french notation). By this result and our variant of the KRS- correspondence we are able to infer the linear independence of the *c-arrays* modulo I_w in the multilinear case. Further combinatorial arguments allow to extend the independence to the multihomogeneous case. All computations and algorithms have been implemented by *Maple*.

SPECHT' PROBLEM IN POSITIVE CHARACTERISTIC

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In this lecture we are going to treat various aspects of the famous Specht's Problem in zero and positive characteristic. We will discuss both the classical formulation of the problem in the language of T -ideals and its generalization in the modern language of T -spaces. Our aim is the most recent results on Specht's Problem achieved by a number of mathematicians.

ALMOST POLYNOMIAL GROWTH VARIETIES OF LINEAR ALGEBRAS

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We study the behaviour of codimension sequence of polynomial identities of linear algebras over a field F of characteristic 0. We give known and new results about linear algebras varieties with almost polynomial growth in the case of associative (with or without involution, \mathbf{Z}_2 -graded), Lie and Leibniz algebras. In the theory of associative algebras the basic role in this area play the infinite dimensional Grassmann algebra G and the algebra of 2×2 upper triangular matrices UT_2 . Well known that there are only two varieties of associative algebras with almost polynomial growth. They are generated by algebras G and UT_2 respectively. The name of author is A.Kemer. In the theory of associative algebras with involution the situation is the similar. There exist only two varieties with almost polynomial growth. The first is the algebra $G_2 = F \oplus F$ with exchange involution, that plays a role analogous to the role played by the infinite dimensional Grassmann algebra in the ordinary theory. The next herouz is the some four dimensional algebra that plays a role analogous to the role played by the algebra of 2×2 upper triangular matrices in the ordinary theory. The names of authors are A.Giambruno, S.Mishchenko and A.Valenty. In the theory of associative \mathbf{Z}_2 -graded algebras there exist only five varieties with almost polynomial growth. Basic role play the infinite dimensional Grassmann algebra with trivial or natural grading; the algebra 2×2 upper triangular matrices with trivial or natural grading. The fifth variety generated by commutative algebra $A = F \oplus tF$ where $t^2 = 1$ with grading $A_0 = F$, $A_1 = tF$. The names of authors are A.Gambruno, S.Mishchenko, A.Valenty and M.Zaicev. In the theory of varieties of Lie algebras there exist only four solvable varieties with almost polynomial growth. So far only one example of a non solvable variety with almost polynomial growth has been constructed: this is the variety gener-

ated by the 3-dimensional simple Lie algebra. The existence of new examples of non soluble varieties with almost polynomial growth seems an interesting and hard problem in the theory of varieties of Lie algebras is very hard problem. The names of authors are V.Drensky, S.Mishchenko, Yu.Razmuslov and I.Volichenko. By definition an algebra is Leibniz algebra if the product from right side is the derivation. The correspond identity has the form

$$(xy)z \equiv (xz)y + x(yz).$$

Similar to Lie case the author and L.Abanina (Ulyanovsk state university, Russia) construct three new examples of the varieties with almost polynomial growth. These three varieties are the similar to the four solvable Lie algebras varieties but one pair from the Lie case give only one new example of Leibniz algebras variety with almost polynomial growth. Give more information about the last one. Denote by $\widetilde{M}_2 = \{h, e | he = h\}$ two-dimensional Leibniz algebra (the other product of basis elements equal to zero. Let R be an associative commutative ring of polynomials in the variable t consider as algebra with zero table of multiplication. We turn R into an \widetilde{M}_2 -module by setting

$$f(t)h = tf(t), f(t)e = tf'(t), hf(t) = ef(t) = 0,$$

where the prime on the polynomial denomes the taking of the derivative. Let \widetilde{M} be direct product of vector spaces R and \widetilde{M}_2 . Define the product as

$$(m_1 + f_1)(m_2 + f_2) = g_1g_2 + f_1g_2,$$

where $m_1, m_2 \in \widetilde{M}_2, f_1, f_2 \in R$. The variety $var \widetilde{M}$ generated by algebra \widetilde{M} has almost polynomial growth.

PROPER SUBVARIETIES OF \mathfrak{M}_5

Vincenzo Nardoza

Italy

Let \mathbb{K} be a field of characteristic zero, and \mathfrak{M}_5 the variety of associative unitary algebras generated by the polynomial identity $[x_1, x_2][x_3, x_4, x_5]$. We describe asymptotically its lattice of the proper subvarieties. More precisely, we define certain algebras \mathcal{R}_{2k} for any $k \in \mathbb{N}$ and show that if \mathfrak{U} is a proper subvariety of \mathfrak{M}_5 , then its T -ideal is asymptotically equivalent to the T -ideal of \mathbb{K} , E , \mathcal{R}_{2k} or $\mathcal{R}_{2k} \oplus E$, for a suitable $k \in \mathbb{N}$.

POLYNOMIAL IDENTITIES AND REDUCIBILITY OF SEMIGROUPS

Heydar Radjavi
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We are interested in the effect of polynomial identities on reducibility and triangularizability of multiplicative semigroups of linear operators. We shall confine ourselves to finite dimensions, although our questions can also be asked and answered for compact operators on Banach spaces. For a specific case, let S denote a semigroup of operators on a finite-dimensional space and consider the question: What polynomials f in two noncommuting variables have the property that if some power of f is identically zero on S , then S is simultaneously triangularizable? A known example of such a polynomial, given by a theorem of Guralnick, is $f(x, y) = xy - yx$. We discuss several results, including generalizations of this theorem, and some examples of interest.

A POLYNOMIAL PROBLEM ARISING FROM T -DESIGNS

Rolf Rees

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A t -wise balanced design of type $t - (v, K, \lambda)$ is a pair (X, B) where X is a v -element set of points and B is a collection of sub-sets of X called blocks such that every t -element subset of X is contained in exactly λ blocks and the number of points in any given block is a member of K . In consideration of a problem concerning the existence of a certain class of t -designs there arises a sequence of polynomials in two variables for which we can determine a recurrence relation but not a closed-form formula. Our interest is in the behavior of this class of polynomials; in particular, we would like to determine for which $x, y \geq 0$ these polynomials take on non-negative values. This talk constitutes a desperate plea for help!

EXPLICIT DECOMPOSITION OF THE GROUP ALGEBRA FA_N OF THE ALTERNATING GROUP A_N

Amitai Regev

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In talk 1 we describe the decomposition of the group-algebra of the alternating group into a direct sum of simple two sided ideals, then the decomposition of each such ideal into a direct sum of simple left ideals. The description is in terms of Young tableaux, and is similar to the well known analogue decomposition of the group-algebra FS_n of the symmetric group S_n .

Talk 2. A_n -codimensions and cocharacters. The fact that FA_n is a sub-space of FS_n gives rise to FA_n codimensions and cocharacters. We study these invariants for various p.i. algebras, in particular for the Grassmann algebra. Joint work with A. Henke.

COMPUTING WITH MATRICES OVER THE GRASSMANN ALGEBRA

Jeno Szigeti

*University of Miskolc
Hungary)*

We shall consider $n \times n$ matrices over a Grassmann algebra part of A . Let v_i be a generator of E not occurring in the elements of A , then A can be completely read off the so called companion matrix $A_0 + A_1 v_i$ and $A_0 + A_1 v_i \in M_n(E_0)$ lies in a commutative environment. The main aim of the talk is to show, how can we use $A_0 + A_1 v_i$ instead of A in different calculations. More generally, for $n \times n$ matrices, over a \mathbb{Z}_2 -graded ring $R = R_0 \oplus R_1$ with $R_0 \subseteq Z(R)$, our computational technique leads to a new concept of determinant, which can be used to derive an invariant Cayley-Hamilton identity. This talk reports a joint work with Sudarshan Sehgal.

GRADINGS AND GRADED IDENTITIES OF THE ALGEBRA OF $N \times N$ UPPER TRIANGULAR MATRICES

Angela Valenti
University of Palermo

We study the graded identities of the algebra UT_n of $n \times n$ upper triangular matrices over a field F endowed with an elementary grading. Such gradings appear quite naturally. For instance it has been recently proved that every finite abelian grading on the algebra UT_n is elementary provided F is algebraically closed of characteristic zero.

In case F is infinite and UT_n has its usual \mathbf{Z}_n -grading, we describe a basis of the ideal of the graded identities for this algebra and we compute some of the numerical invariants of this ideal. In general we show that one can distinguish the elementary gradings by means of the graded identities they satisfy.

NUMERICAL CHARACTERISTICS AND EXTREMAL VARIETIES

Mikhail Zaicev

Moscow State University

We consider varieties of associative algebras over a field of characteristic zero. Dimension of multilinear component in N variables of the relatively free algebra of the variety is usually called its N -th codimension. An important numerical characteristics of a variety is the sequence of codimensions and its asymptotic behaviour. It is well-known that any variety of associative algebras over a field of characteristic zero has exponentially bounded codimension growth. Recently it was proved that the exponent of any variety exists and is an integer. We say that V is a minimal variety of given exponent n if $\exp(V)=n$ and $\exp(W)<n$ for any proper subvariety W of V . We describe all minimal varieties of non-polynomial growth in terms of generating algebras and T -ideals. For example, any minimal variety of a finite basic rank can be generated by an upper block-triangular matrix algebra. In particular, we prove in the positive a conjecture of Drensky that V is a minimal variety if and only if its T -ideal is the product of verbally prime T -ideals. As a corollary we obtain that there is only a finite number of minimal varieties for any given exponent.