Atlantic Association for Research in the Mathematical Sciences

Memorial University of Newfoundland

Atlantic Algebra Centre

CRG Groups, Rings, Lie and Hopf Algebras

International Workshop

Groups, Rings, Lie and Hopf Algebras. V

Aug 21 - Aug 25, 2023

Memorial University of Newfoundland
Harlow, UK

Schedule and Abstracts of Talks

Organizing committee

Yuri Bahturin

Mikhail Kochetov

Kirill Zaynullin

Alexander Premet

St. John's, NL
2023

Monday, Aug 21, 2023

8:30	-	9:00	**Opening/Registration**
9:00	-	9:50	**Alberto Elduque** *University of Zaragoza, Spain* Gradings and S-structures on Lie algebras
10:00	-	10:50	**Yuly Billig** *Carleton University, Canada* Sheaves of AV-modules over projective varieties
11:00	-	11:30	**Coffee break**
11:30	-	12:20	**Michael Vaughan-Lee** *University of Oxford, UK* Computing with finite p-groups and LiePRings
12:30	-	12:50	**Inga Valentiner-Branth** *Ghent University, Belgium* High Dimensional Expanders from Kac-Moody-Steinberg Groups
1:00	-	2:00	**Lunch Break**
2:00	-	2:50	**Gunnar Traustason** *University of Bath, UK* Left 3-Engel elements in groups
3:00	-	3:20	**Tom Baird** *Memorial University of Newfoundland, Canada* Cohomology of fixed point Lagrangians in the Hilbert scheme of points of a symplectic surface
3:30	-	4:00	**Coffee Break**
4:00	-	4:30	**Sam Hughes** *University of Oxford, UK* Finite quotients of generic free-by-cyclic groups
4:40	-	5:00	**Shadi Shaqaqha** *Yarmouk University, Jordan* Hom-Lie algebras and related topics

Tuesday, Aug 22, 2023

9:00	-	9:50	**Eli Aljadeff** *Technion, Israel* On Semisimple PI invariants of finite dimensional algebras
10:00	-	10:50	**Onofrio Mario Di Vincenzo** *Università degli Studi della Basilicata, Italy* On minimal varieties of graded PI-algebras by abelian groups
11:00	-	11:30	**Coffee break**
11:30	-	12:20	**Rostislav Grigorchuk** *Texas A & M University* Branch groups and their subgroups
12:30	-	12:50	**Sebastiano Argenti** *Università degli Studi della Basilicata, Italy* Lie semisimple algebras of derivations and varieties of almost polynomial growth
1:00	-	2:00	**Lunch Break**
2:00	-	2:50	**Erhard Neher** *University of Ottawa, Canada* Quadratic forms over rings
3:00	-	3:20	**Cameron Ruether** *Memorial University of Newfoundland, Canada* The Norm Functor over Schemes
3:30	-	4:00	**Coffee Break**
4:00	-	4:30	**Ashot Minasyan** Trace submonoids of 1-relator groups *University of Southampton, UK* Trace submonoids of 1-relator groups
4:40	-	5:00	**Bryan Kettle** *University of Alberta, Canada* Orthosymplectic Yangians

Wednesday, Aug 23, 2023

9:00	-	9:50	**Tomasz Brzezinski** *Swansea University, UK* Associative and Lie algebras on affine spaces or trusses and Lie affgebras
10:00	-	10:30	**Yorck Sommerhäuser** *Memorial University of Newfoundland, Canada* Yetter-Drinfel'd Hopf Algebras and Their Extensions
10:40	-	11:00	**Coffee break**
11:00	-	11:50	**Christoph Schweigert** *Universität Hamburg, Germany* Davydov-Yetter, comonad and relative cohomology: Remarks on rigidity and on deformations
12:00	-	1:00	**Lunch Break**

Thursday, Aug 24, 2023

9:00	-	9:50	**Eric Jespers** *Vrije Universiteit Brussel, Belgium* Quadratic algebras and set-theoretic solutions of the Yang–Baxter equation
10:00	-	10:50	**Pere Ara** *Universitat Autònoma de Barcelona, Spain* Inverse semigroups of separated graphs and their associated algebras
11:00	-	11:30	**Coffee break**
11:30	-	12:20	**Seidon Alsaody** *Uppsala Universitet, Sweden* Brown algebras and groups of type E6 and E7 over rings
12:30	-	1:30	**Lunch Break**
1:30	-	2:20	**Agata Smoktunowicz** *University of Edinburgh, UK* A look at contemporary research into nil and nilpotent rings
2:30	-	3:20	**Susan Sierra** *University of Edinburgh, UK* Enveloping algebras of infinite-dimensional Lie algebras
3:30	-	4:00	**Coffee Break**
4:00	-	4:30	**Alexander Baranov** *University of Leicester, UK* TBA
4:40	-	5:00	**Arne Van Antwerpen** *Vrije Universiteit Brussel, Belgium* Cabling non-involutive set-theoretic solutions of the Yang-Baxter equation

Friday, Aug 25, 2023

9:00	-	9:50	**Juan Cuadra** *Universidad de Almería, Spain* Non-existence of integral Hopf orders for twists of several simple groups of Lie type
10:00	-	10:50	**Ehud Meir** *University of Aberdeen, UK* Invariants that are covering spaces and their Hopf algebras
11:00	-	11:30	**Coffee break**
11:30	-	12:20	**Sergey Shpectorov** *University of Birmingham, UK* Solid subalgebras and the geometry of axes in algebras of Jordan type half
12:30	-	1:30	**Lunch Break**
1:30	-	2:20	**Michael Lau** *Université Laval, Canada* Toda Systems for Takiff Algebras
2:30	-	3:00	**Coffee Break**
3:00	-	3:30	**Mitja Mastnak** *Saint Mary's University, Canada* On Hopf algebras whose coradical is an abelian cleft extension
3:40	-	4:10	**Sebastian Burciu** *Institute of Mathematics of the Romanian Academy, Romania* On the zero entries in the character table of a fusion category

Eli Aljadeff

Technion, Israel

On Semisimple PI invariants of finite dimensional algebras

Kemer showed that if W is an associative affine PI algebra over a field of characteristic zero then it is PI equivalent to a finite dimensional algebra B. More generally, the same holds if W satisfies a Capelli identity. In case W does not satisfy any Capelli identity the finite dimensional algebra B is replaced by the Grassmann envelope of a finite dimensional super algebra B_2.

It is well known (and easy to show) that the finite dimensional algebra B (and similarly the super algebra B_2) is not determined by its T-ideal of identities. The purpose of this lecture is to show that the semisimple part of B (resp. of B_2) is "basically" determined by the ideal of identities. This determines a semisimple algebra (or super algebra) invariant for any T-ideal. In the lecture I will explain what "basically" means here. Based on time I will present generalizations of this result to the group graded setting. Joint work with Karasik.

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Seidon Alsaody

Uppsala Universitet, Sweden

Brown algebras and groups of type E6 and E7 over rings

Brown algebras are 56-dimensional structurable algebras. Classically studied over fields of characteristic different from 2 and 3, their automorphism groups are algebraic groups of type E6. To each Brown algebra one can associate a Freudenthal triple system, whose invariance group is an algebraic group of type E7.

It is natural to ask how Brown algebras can be defined over general (unital, commutative) rings of scalars, in the interest of studying group schemes of type E6 and E7 over an arbitrary base. I will report on recent results aiming to answer this question. I will define the abovementioned concepts, including a recent formulation of Freudenthal triple systems, due to Garibaldi, Petersson and Racine, in a general setting. When 2 (and sometimes 3) is required to be invertible in the ring of scalars, the homogeneous space E7/E6 is seen to parametrise isotopes of Brown algebras, unveiling phenomena that do not appear over fields.

Pere Ara

Universitat Autònoma de Barcelona, Spain

Inverse semigroups of separated graphs and their associated algebras

A separated graph is a pair (E, C), where E is a directed graph, $C = \bigsqcup_{v \in E^0} C_v$, and C_v is a partition of $s^{-1}(v)$ (into pairwise disjoint nonempty subsets) for every vertex v. For each separated graph (E, C), we will introduce its inverse semigroup $\mathcal{S}(E, C)$ and we will give a normal form for its elements. We will relate this inverse semigroup with some tame algebras associated to (E, C), such as the tame Cohn path algebra $C^{\mathrm{ab}}(E, C)$ and the tame Leavitt path algebra $L^{\mathrm{ab}}(E, C)$.

This is joint work in progress with Alcides Buss and Ado Dalla Costa, both from Universidade Federal de Santa Catarina.

Sebastiano Argenti

Università degli Studi della Basilicata, Italy

Lie semisimple algebras of derivations and varieties of almost polynomial growth

In this talk we are interested in the growth of the differential identities of finite dimensional associative algebras, i.e., polynomial identities of algebras with an action of a Lie algebra by derivations.

In this context Gordienko and Kochetov proved that, over fields of characteristic zero, the exponent always exists and is a positive integer. As a consequence of their proof, the codimension sequence associated of the differential identities either grows polynomially or exponentially [1].

We are interested in characterizing associative algebras A of almost polynomial growth, that is the codimension sequence of A grows exponentially but for any finite dimensional algebra in the variety generated by A, generating a proper subvariety, the corresponding growth is only polynomial.

This problem appears naturally in the study of varieties of PI-algebras with (or without) some additional structure and was solved for associative algebras [2], graded algebras [3], algebras with involution [4], etc...

A common flavor of these results is the existence of a finite number of such varieties and the indication of a concrete list of generating algebras. Our main result concerns the characterization of varieties of almost polynomial growth in case the Lie algebra acting by derivations is $\mathfrak{sl}_2$. In this case we obtain two infinite classes of algebras generating distinct differential varieties of almost polynomial growth.

We also discuss the more general case in which L is any finite dimensional semisimple Lie algebra. We exhibit a list of algebras such that the codimension growth of a variety is exponential if and only if it contains one of the algebras in the list.

References

[1] A. S. Gordienko, M. V. Kochetov, Derivations, gradings, actions of algebraic groups, and codimension growth of polynomial identities. Algebras and Representation Theory, 17(2):539-563, mar 2014.

[2] A. R. Kemer, Nonmatrix varieties. (Russian) Algebra i Logika 19 (1980), no. 3, 255–283, 382.

[3] A. Valenti, Group graded algebras and almost polynomial growth. J. Algebra 334 (2011), 247–254.

[4] A. Giambruno; S. Mishchenko, On star-varieties with almost polynomial growth. Algebra Colloq. 8 (2001), no. 1, 33–42.

Tom Baird

Memorial University of Newfoundland, Canada

Cohomology of fixed point Lagrangians in the Hilbert scheme of points of a symplectic surface

Let S be a smooth, quasi-projective complex surface with complex symplectic form $\omega \in H^0(S, K_S)$. This determines a symplectic form ω_n on the Hilbert scheme of points $S^{[n]}$ for $n \geq 1$. Let τ be a symplectic involution of (S, ω): an order two automorphism of S such that $\tau^*\omega = -\omega$. Then τ induces an anti-symplectic involution on $(S^{[n]}, \omega_n)$. The fixed point set $(S^{[n]})^\tau$ is a smooth Lagrangian subvariety of $S^{[n]}$. In this paper, we calculate the mixed Hodge structure of $H^*((S^{[n]})^\tau; Q)$ in terms of the mixed Hodge structures of $H^*(S^\tau; Q)$ and of $H^*(S//\langle\tau\rangle; Q)$. Our proof adapts an argument of Göttsche-Soergel: we prove that the restricted Hilbert-Chow morphism $(S^{[n]})^\tau \to (S^{(n)})^\tau$ is semi-small and apply the Decomposition Theorem of Beilinson-Bernstein-Deligne-Gabber.

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Yuly Billig

Carleton University, Canada

Sheaves of AV-modules over projective varieties

AV-modules are representations of Lie algebra V of vector fields that admit a compatible action of the commutative algebra A of functions. This notion is a natural generalization of D-modules. In this talk we shall start by reviewing the theory of AV-modules over smooth irreducible affine varieties. When variety X is projective, it is necessary to consider sheaves of AV-modules. We describe associative algebras that control the category of AV-modules, and construct a functor from the category of strong representations of Lie algebra of jets of vector fields to the category of AV-modules. This talk is based on the joint work with Colin Ingalls, as well as the work of Emile Bouaziz and Henrique Rocha.

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Tomasz Brzezinski

Swansea University, UK

Associative and Lie algebras on affine spaces or trusses and Lie affgebras

There is a natural way of viewing an affine space as a set with a ternary operation and (also ternary) multiplication by scalars. This point of view removes an underlying vector space and allows for the formulation of affine maps with no reference to linear transformations or any specific points in the affine space. Considering associative bi-affine multiplications leads to a generalised distributive law which includes the usual distributivity of multiplication over addition and the brace-type distributive law as special cases. In short one needs to require that multiplication distributes over the ternary operation naturally defined on an affine space; the resulting ring-type structure is known as a truss. Similarly, the intrinsic point of view on Lie structures on an affine space requires a modification of the axioms of a Lie algebra, leading to a notion of a Lie affgebra (a term coined in this context by K. Grabowska, J. Grabowski and P. Urbański in the early 2000s). Apart from giving the definition of trusses and Lie affgebras and providing a motivation for their introduction I will describe their properties by comparing and contrasting them with basic properties of rings and Lie algebras.

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Sebastian Burciu

Institute of Mathematics of the Romanian Academy, Romania

On the zero entries in the character table of a fusion category

In this talk we extend a classical vanishing result of Burnside from the character tables of finite groups to the character tables of fusion categories or more generally to a large class of commutative fusion rings. We also consider the dual vanishing result. As applications, we prove new identities that hold in the Grothendieck ring of any weakly-integral fusion category satisfying the dual Burnside vanishing result. Based on a joint work with Sebastien Palcoux.

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Juan Cuadra

Universidad de Almería, Spain

Non-existence of integral Hopf orders for twists of several simple groups of Lie type

In [1] and [2], we found an arithmetic difference between group algebras and semisimple Hopf algebras in connection with Kaplansky's sixth conjecture. Namely, *there are complex semisimple Hopf algebras that do not admit an integral Hopf order.* The families of examples for which this phenomenon occurs turn out to be simple Hopf algebras. The following question was raised in [2]:

Let G be a finite group and Ω a non-trivial twist for $\mathbb{C}G$, arising from an abelian subgroup, such that the twisted Hopf algebra $(\mathbb{C}G)_\Omega$ is simple. Can $(\mathbb{C}G)_\Omega$ admit an integral Hopf order?

In this talk, we will answer this question in the negative for the projective special linear groups of order 2 and 3 and for the Suzuki groups. This, reinforced with two theorems on minimal simple groups, will allow us to give the following partial answer: for any finite non-abelian simple group G there is a twist for $\mathbb{C}G$ such that the twisted Hopf algebra $(\mathbb{C}G)_\Omega$ does not admit an integral Hopf order.

This talk is based on the paper [3] in collaboration with Giovanna Carnovale and Elisabetta Masut from the University of Padova, Italy.

References

[1] J. Cuadra and E. Meir, *On the existence of orders in semisimple Hopf algebras.* Trans. Amer. Math. Soc. 368 (2016), 2547-2562.

[2] ______, *Non-existence of Hopf orders for a twist of the alternating and symmetric groups.* J. London Math. Soc. (2) 100 (2019) 137-158.

[3] G. Carnovale, J. Cuadra, and E. Masut, *Non-existence of integral Hopf orders for twists of several simple groups of Lie type.* To appear in Publ. Mat. ArXiv:2108.12324.

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Onofrio Mario Di Vincenzo

Università degli Studi della Basilicata, Italy

On minimal varieties of graded PI-algebras by abelian groups

In the theory of polynomial identities of associative algebras over a field F of characteristic zero a relevant role is played by the sequence of codimensions of the P.I.-algebra A, $\{c_n(A)\}_{n\geq 1}$, introduced by Regev in [5]. A classic result of Giambruno and Zaicev [2] states that

$$exp(A) = \lim_{n\to\infty} \sqrt[n]{c_n(A)}$$

always exists and it is a non negative integer called the PI-exponent of the algebra A. As usual in this context, we can refer to the variety $\mathcal{V} = var(A)$ generated by A, and we consider the sequence $\{c_n(\mathcal{V})\}_{n\geq 1}$ and also the PI-exponent, $exp(\mathcal{V})$, of a given variety $\mathcal{V}$. In particular, $\mathcal{V}$ is said minimal of exponent d if $exp(\mathcal{V}) = d$ and $exp(\mathcal{U}) < d$ for all its proper subvarieties, [3].

In [1] Aljadeff and Giambruno proved that the graded exponent $exp_G(A)$ of a G-graded PI-algebra A always exists and it is an integer for all finite groups G. As in the ordinary case, the existence of the corresponding exponent allows to classify varieties of G-graded PI-algebras on an integral scale whose steps are the minimal varieties of given exponent d, [4].

In this talk we consider the case when the group G is abelian. More precisely, given a sequence of finite dimensional G-simple algebras $A_1, \ldots, A_m$ we define a block triangular matrix algebra $UT(A_1, \ldots, A_m, \gamma)$ equipped with an elementary G-grading, having $A_1, \ldots, A_m$ as blocks on the diagonal.

We prove that an affine variety $\mathcal{V}$ of G-graded PI-algebras is minimal if and only if it is generated by a G-graded algebra $UT(A_1, \ldots, A_m, \gamma)$ defined over a suitable field K, finite extension of F, and satisfying $dim_K A_1 + \ldots + dim_K A_m = exp_G(\mathcal{V})$.

This is a joint work with S. Argenti.

References

[1] E. Aljadeff, A. Giambruno *Multialternating graded polynomials and growth of polynomial identities*. Proc. Amer. Math. Soc. 141(9):3055-3065, 6 (2013).

[2] A. Giambruno, M. Zaicev, *Exponential codimension growth of PI algebras: an exact estimate*. Adv.Math. 142(2):221-243, (1999).

[3] A. Giambruno, M. Zaicev, *Codimension growth and minimal superalgebras*. Trans. Amer. Math. Soc. 355(12):5091-5117, (2003).

[4] O. M. Di Vincenzo, V. R. T. da Silva, E. Spinelli, *A characterization of minimal varieties of $\mathbb{Z}_p$-graded PI algebras*. Journal of Algebra, 539: 397-418, (2019).

[5] A. Regev. *Existence of identities in $A\otimes B$*. Israel J Math, 11(2):131- 152, 6 (1972).

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Alberto Elduque

University of Zaragoza, Spain

Gradings and S-structures on Lie algebras

A few years ago, Vinberg considered a broad extension of gradings by abelian groups on Lie algebras. In this talk the S-structures of Vinberg will be reviewed, examples that had already appeared in the literature and that fit in this framework will be given, and some new examples, obtained in joint work with P. Beites, A. Córdova-Martínez, and I. Cunhá, will be provided.

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Sam Hughes

University of Oxford, UK

Finite quotients of generic free-by-cyclic groups

In this talk we will investigate to what extent we can distinguish two free-by-cyclic groups by their finite quotients. I will present recent work joint with Monika Kudlinska proving that there exist at most finitely many free-by-cyclic groups with the same finite quotients as a generic free-by-cyclic group.

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Rostislav Grigorchuk

Texas A & M University

Branch groups and their subgroups

I will begin with a short account of the history and current state of the theory of groups of branch type. Then I will focus on the subgroup structure of branch groups, starting from a joint work with J.S. Wilson and ending with recent results with D. Francoeur, P-H. Leemann and T. Nagnibeda concerning the existence of the block structure in finitely generated subgroups. The exposition will mostly concern the abstract branch groups, but the profinite version will also be mentioned, as well as the Lie algebras associated to the branch groups of intermediate growth.

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Eric Jespers

Vrije Universiteit Brussel, Belgium

Quadratic algebras and set-theoretic solutions of the Yang–Baxter equation

Given a set X and a field K, one associates to a map $r : X \times X \to X \times X$, with $(x,y) \mapsto (\lambda_x(y), \rho_y(x))$, a quadratic algebra $\mathcal{A}_K(X,r) = K\langle X \mid xy - \lambda_x(y)\rho_y(x),\ x,y \in X\rangle$. This algebra often is called the structure algebra of (X,r). It has a presentation defined by quadratic word relations and thus it is the monoid algebra of the structure monoid $M(X,r)$, i.e. the monoid generated by the set X and subject to the "same" word relations. This algebra is the associative ring theoretic tool to investigate the map r and has attracted a lot of attention in case r satisfies the braided relation, i.e. (X,r) is a set-theoretic solution of the Yang-Baxter equation. Fundamental results have been proven for algebras of such solutions. For example, if X is finite and r is bijective and non-degenerate, i.e. all maps λ_x and ρ_y are bijective, then the algebra satisfies a polynomial identity and is left and right Noetherian and has finite Gelfand-Kirillov dimension. If, furthermore, r is involutive then these algebras share many properties with polynomial algebras in commuting variables.

The aim of this lecture is to explain the intriguing relationship between the algebraic structure of the structure algebras $\mathcal{A}_K(X,r)$ and the finite left non-degenerate (i.e. all maps λ_x are bijective and X is finite) set-theoretic solutions (X,r) of the Yang-Baxter equation. The main focus is on when such algebras are Noetherian, prime, semiprime, representable and the Gelfand-Kirillov dimension.

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Bryan Kettle

University of Alberta, Canada

Orthosymplectic Yangians

Yangians form one of the two important families of affine quantum groups, alongside the Drinfel'd-Jimbo quantum affine algebras. Given a complex simple Lie algebra or non-exceptional basic Lie superalgebra $\mathfrak{g}$, the Yangian $Y(\mathfrak{g})$

is a certain Hopf algebra which is a deformation of the universal enveloping algebra $U(\mathfrak{g}[z])$. In this talk, we cover recent advancements made on the structure and representation theory of Yangians based on the orthosymplectic Lie superalgebras.

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Michael Lau

Université Laval, Canada

Toda Systems for Takiff Algebras

Toda systems were introduced in the late 1960s as integrable lattice models for nonlinear particle interactions. Their Hamiltonians can be reinterpreted in terms of root systems, with coadjoint orbits serving as classical phase spaces. After an introduction from first principles, we will discuss how to extend and exactly solve such integrable systems in the context of Takiff algebras, Lie algebras of matrices over a ring of (truncated) polynomials of bounded degree. Such algebras have appeared in a number of applications including logarithmic CFT and jet schemes. The resulting Hamiltonians have potentials which are products of polynomial and exponential functions. We close with some remarks on lifting the classical systems to families of commuting operators in the corresponding universal enveloping algebras.

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Mitja Mastnak

Saint Mary's University, Canada

On Hopf algebras whose coradical is an abelian cleft extension

This is joint work with Gaston Garcia. We study the structure of Hopf algebras whose coradical is a semisimple Hopf algebra K_n obtained as a cocentral extension of a group algebra by a dual group algebra. We describe the simple Yetter-Drinfeld modules, compute the fusion rules and determine the finite-dimensional Nichols algebras for some of them. In particular, the well-known

Fomin-Kirillov algebras appear as Nichols algebras over K_3. As a byproduct we obtain new Hopf algebras of dimension 216.

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Ehud Meir

University of Aberdeen, UK

Invariants that are covering spaces and their Hopf algebras

Different flavours of string diagrams arise naturally in studying algebraic structures (e.g. algebras, Hopf algebras, Frobenius algebras) in monoidal categories. In particular, closed diagrams can be realized as scalar invariants. For a structure of a given type the closed diagrams form a commutative algebra that has a richer structure of a self dual Hopf algebra. This is very similar, but not quite the same, as the positive self adjoint Hopf algebras that were introduced by Zelevinsky in studying families of representations of finite groups. In this talk I will show that the algebras of invariants admit a lattice that is a PSH-algebra. This will be done by considering maps between invariants, and realizing them as covering spaces. I will then show some applications to subgroup growth questions, and a formula that relates the Kronecker coefficients to finite index subgroups of free groups.

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Ashot Minasyan

University of Southampton, UK

Trace submonoids of 1-relator groups

Given a finite simplicial graph Γ the trace monoid (a.k.a. partially commutative monoid) $T(\Gamma)$ associated to this graph is the monoid generated by the vertices of Γ subject to the relations that two vertices commute if and only if they are adjacent in Γ. A group with the same presentation is called the right angled Artin group $A(\Gamma)$. It is known that $A(\Gamma)$ contains $T(\Gamma)$ as its submonoid of positive words.

Trace monoids originated in Computer Science, but more recently they have been used to establish certain undecidability results for 1-relation inverse monoids and groups. On the other hand, right angled Artin groups play an important role in Geometric Group Theory.

In a recent work, Foniqi, Gray and Nyberg-Brodda showed that groups containing $T(P_4)$, where P_4 is the path with 4 vertices (of length 3), have undecidable rational subset problem. They also exhibited 1-relator groups containing $A(P_4)$ and asked whether every 1-relator group which has a submonoid isomorphic to $T(P_4)$ must also have a subgroup isomorphic to $A(P_4)$. In my talk I will discuss joint work with Motiejus Valiunas showing that the answer to the latter question is positive.

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Erhard Neher

University of Ottawa, Canada

Quadratic forms over rings

We give an introduction to quadratic forms over rings by describing an extension of a classical result from the theory of quadratic forms over fields, namely Springer's Odd Degree Extension Theorem, to quadratic forms over semilocal rings. The talk is based on joint work with Philippe Gille.

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Cameron Ruether

Memorial University of Newfoundland, Canada

The Norm Functor over Schemes

The corestriction with respect to a finite separable field extension L/F, which sends L-vector spaces to F-vector spaces, defined by Riehm was one the first instance of the norm functor studied. The notion was extended by other authors such as Gabber as well as Knus and Ojanguren to the setting of finite étale extension of rings. In 1998, Ferrand further extended the norm to finite locally free extensions of rings. We will introduce Ferrand's norm functor, reviewing its main properties, before discussing a recent further generalization which is joint work with Philippe Gille and Erhard Neher. For a finite locally free covering of schemes we define a norm functor on quasi-coherent modules which is most naturally described as a stack morphism. We then discuss the cohomological interpretation of this norm and describe how it is related to the Segre (or sometimes called Kronecker) embedding of general linear and projective general linear groups. Using this cohomological description, we show that the norm morphism can be restricted to provide an equivalence of stacks between objects of type $A_1 \times A_1$ and those of type D_2 in this setting, as the isomorphism of Dynkin diagrams would suggest. This result generalizes the equivalence of groupoids result present in The Book of Involutions, which uses the norm functor for étale extensions of a field.

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Christoph Schweigert

Universität Hamburg, Germany

Davydov-Yetter, comonad and relative cohomology: Remarks on rigidity and on deformations

We explain how various deformation problems for monoidal categories, monoidal functors and module categories lead to Davydov-Yetter (DY) cocycles, once one introduces coefficients for DY-cohomology. Via comonad cohomology for suitable monad, DY cohomology can be expressed as relative cohomology. These reformulations provide a conceptual understanding of Ocneanu rigidity and powerful tools for the computation of Davydov-Yetter cohomologies.

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Shadi Shaqaqha

Yarmouk University, Jordan

Hom-Lie algebras and related topics

Hom-Lie algebras have emerged as a significant area of study in nonassociative algebraic structures, offering a generalization of Lie algebras. This presentation delves into the exploration of regular and multiplicative Hom-Lie algebras, showcasing a diverse range of new examples to illustrate their intriguing structure. Remarkably, we establish that any Hom-Lie algebra L defined over a field with characteristic $\neq 2$ possesses a bracket operation that endows it with the structure of a Lie algebra. Expanding our investigation, we delve into the realm of infinite direct products of Hom-Lie algebras, unveiling noteworthy properties associated with this construction. Furthermore, we establish the satisfaction of isomorphism theorems for Lie algebras within the realm of Hom-Lie algebras.

Building upon this foundation, we explore solvable and nilpotent Hom-Lie algebras, drawing inspiration from the well-developed theory of solvable and nilpotent groups in group theory. By establishing an analogous theory for Hom-Lie algebras, we contribute to a deeper understanding of these algebraic structures. Supporting our findings with illustrative examples, we identify potential avenues for future research in this area. Additionally, we introduce the concept of fuzzy Hom-Lie subalgebras and ideals, expanding our exploration further. Our primary objective is to examine their properties and investigate the relationship between fuzzy Hom-Lie subalgebras/ideals and their conventional counterparts. We demonstrate the construction of new fuzzy Hom-Lie subalgebras by leveraging the direct sum of existing ones. Furthermore, we analyze the properties of fuzzy Hom-Lie subalgebras and fuzzy Hom-Lie ideals within the context of morphisms of Hom-Lie algebras. This talk sheds light on the intricate structure of Hom-Lie algebras, delves into solvability theory, and explores the captivating realm of fuzzy subalgebras. By addressing these various aspects, we pave the way for further advancements in the field, fostering a deeper understanding of Hom-Lie algebras and their applications.

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Sergey Shpectorov

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Solid subalgebras and the geometry of axes in algebras of Jordan type half

This is based on a joint project with Ilya Gorshkov and Alexey Staroletov.

The class of algebras of Jordan type η was introduced by Hall, Rehren and Shpectorov in 2015. They are commutative non-associative algebras generated by primitive idempotents, called axes, forcing a Peirce-type decomposition of the algebra. In the original paper, these algebras were classified for all $\eta \neq \frac{1}{2}$. Namely, it was shown that such an algebra A is either a Matsuo algebra, coming from a group of 3-transpositions, or a factor of a Matsuo algebra. The case of algebras of Jordan type half remains open, although some partial results have been obtained in this case too.

In this project we work with the new concept of a *solid* subalgebra, which is a subalgebra $B \subseteq A$, such that every primitive idempotent from B is an axis of A. We prove a surprising result that subalgebras generated by two axes are almost always solid. As a consequence of this, we obtain the result that, in zero characteristic, if A contains only a finite number of axes then it is a Matsuo algebra or a factor of one.

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Susan Sierra

University of Edinburgh, UK

Enveloping algebras of infinite-dimensional Lie algebras

Universal enveloping algebras of infinite-dimensional Lie algebras are famously difficult to understand; for example, the fundamental question of whether it is possible for such an enveloping algebra to be (left and right) noetherian has been open since first being asked by Amayo and Stewart in 1974.

In the past decade, however, there has been significant progress on these enveloping algebras, starting with the 2013 proof (with Walton) that used noncommutative algebraic geometry to show that the enveloping algebra of the Witt algebra, or centreless Virasoro algebra, is not noetherian. In particular, evidence is accumulating that the two-sided structure of such an enveloping algebra is often tractable. We survey the current state of knowledge on enveloping algebras of infinite-dimensional Lie algebras, with emphasis on two-sided ideals, on related Poisson algebras, and on consequences for representation theory.

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Yorck Sommerhäuser

Memorial University of Newfoundland, Canada

Yetter-Drinfel'd Hopf Algebras and Their Extensions

The definition of a Hopf algebra is meaningful in any quasisymmetric monoidal base category. For ordinary Hopf algebras, this base category is the category of vector spaces. In the category of vector spaces, the tensor product of two algebras becomes again an algebra by interchanging the two middle tensor factors and then multiplying inside the originally given algebras. In other base categories, this interchange operation is replaced by a more general quasisymmetry.

For Yetter-Drinfel'd Hopf algebras, the base category is the category of Yetter-Drinfel'd modules. Yetter-Drinfel'd Hopf algebras appear in the theory of ordinary Hopf algebras when considering the appropriate notion of semidirect product, the so-called Radford biproduct, where one of the two factors is in general not an ordinary Hopf algebra, but rather a Yetter-Drinfel'd Hopf algebra.

Following the now classical theory of group extensions, the extension theory for ordinary Hopf algebras has been developed over a long period of time, starting at least in the sixties of the twentieth century. The corresponding theory for Yetter-Drinfel'd Hopf algebras has a special feature, as in examples the so-called cleaving and cocleaving maps are not morphisms in the base category. In the talk, we give an introduction to Yetter-Drinfel'd Hopf algebras and then cover these aspects of their extension theory.

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Agata Smoktunowicz

University of Edinburgh, UK

A look at contemporary research into nil and nilpotent rings

In this talk I will mention some results about various types of nil rings: Lie rings, pre-Lie rings and noncommutative associative rings. Recall that a ring is nil if each element of this ring to some power is zero. Various types of nilpotency of rings will also be mentioned. We will also look at some connections between nil and nilpotent rings and other research areas as group theory, deformation theory, algebraic geometry and theory of braces and the Yang-Baxter equation, and show how some classical results can be applied in these research areas.

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Gunnar Traustason

University of Bath, UK

Left 3-Engel elements in groups

We survey left 3-Engel elements in groups, focusing in particular on recent developments.

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Inga Valentiner-Branth

Ghent University, Belgium

High Dimensional Expanders from Kac-Moody-Steinberg Groups

Expander Graphs are sparse graphs that are highly connected. This notion has been generalized to simplicial complexes, leading to different definitions of high dimensional expanders. Like the construction of families of expanders using Cayley graphs, we construct new spectral high dimensional expanders using coset complexes and infinite families of finite quotients of Kac-Moody-Steinberg groups. Kac-Moody-Steinberg groups are defined as fundamental groups of certain complexes of finite p-groups, where p is a fixed prime number. The finite p-groups in question are the positive unipotent subgroups of basic Levi subgroups of spherical type in a 2-spherical Kac-Moody group over a finite field.

This rich local structure allows us to describe the links in the coset complex elegantly and use known results on the representation angle of roots subgroups to establish a bound on the local spectral gaps. Using local-to-global methods by Garland and Oppenheim, we show that we constructed infinite families of bounded degree spectral high dimensional expanders.

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Arne Van Antwerpen

Vrije Universiteit Brussel, Belgium

Cabling non-involutive set-theoretic solutions of the Yang-Baxter equation

This talk is based on joint work w. I. Colazzo.

In 1992 Drinfel'd proposed to study set-theoretic solutions (X, r) of the Yang-Baxter equation, i.e. X is a non-empty set and $r : X \times X \longrightarrow X \times X$ satisfies, on X^3, the equation

$$(id_X \times r)(r \times id_X)(id_X \times r) = (r \times id_X)(id_X \times r)(r \times id_X).$$

In this talk we focus on the subclass of bijective non-degenerate solutions.

The cabling of involutive non-degenerate solutions was recently introduced by Lebed, Ramirez and Vendramin, where the technique is used to unify known results, depending on the cycle structure of the square map, on the indecomposability of such solutions. The technique emphasizes heavily the use of the interplay of a secondary group structure on $G(X, r) = gr(x \in X \mid xy = uv \text{ if } r(x, y) = (u, v) >$, i.e. its skew brace structure and that the canonical map $i : X \longrightarrow G(X, r)$ is injective, which fails for non-involutive solutions. We discuss how to extend their method to non-involutive solutions by examining particular subsolutions of the structure monoid $M(X, r) = \langle x \in X \mid xy = uv, \text{ if } r(x, y) = (u, v)\rangle$. Using (semi)group theoretical methods, we will discuss that a bijective non-degenerate solution is indecomposable iff its injectivization, i.e. the natural image in $G(X, r)$ is indecomposable. Moreover, we demonstrate that, under mild conditions, square-free bijective non-degenerate solutions are decomposable. Throughout the talk, we will mention several open problems.

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Michael Vaughan-Lee

University of Oxford, UK

Computing with finite p-groups and LiePRings

The algebraic programming languages GAP and MAGMA contain all sorts of databases which researchers can explore.

The small groups library in MAGMA contains

- The groups of order less than equal to 2000 (excluding 1024). 423,164,062 groups.
- The groups of order p^n $(n \leq 7)$.
- The groups of order 3^8. 1396077 groups.
- The groups of square free order.
- The groups of order p^2q.
- Metacyclic p-groups.

The metacyclic p-groups are not normally considered as part of the small groups library, but I think that it is reasonable to include them.

Only the first and third of these databases contain lists of groups, and the other databases are actually computer programs which generate groups of a given order using a classification of the groups under consideration.

In my talk I give a brief outline of the main tools used in classifying the groups of order p^n $(n \leq 7)$. Actually the classifications of groups of order p^6 and p^7 for $p > 5$ were obtained by classifying the nilpotent Lie rings of order p^6 and p^7 and obtaining the corresponding groups using the Baker-Campbell-Hausdorff formula and the Lazard correspondence between finite p-groups and nilpotent Lie rings.

In 2013 Bettina Eick and I published the GAP package LiePRing which contains a complete classification of the nilpotent Lie rings of order p^n for $p > 3$ and $n \leq 7$. In principle for any such p and n you can obtain a complete list of the nilpotent Lie rings of order p^n. If $p > 5$ then for any Lie ring in the list you can obtain the corresponding group under the Lazard correspondence.

The database contains thousands of generic presentations with symbolic p. For example

$$\langle a, b \,|\, pa - baa,\, pb,\, \text{class} = 4\rangle$$

defines a Lie ring of order p^6 and nilpotency class 4 for any prime p.

There are a number of computations you can do with these generic presentations, leaving the prime p symbolic.

You can work with subrings and ideals with a given set of generators, and compute their presentations, with symbolic p.

You can compute

- The lower central series,
- The lower exponent p central series,
- The derived series,

- The Schur multiplier,
- The p-covering ring.

You can also compute information about the automorphism group with symbolic p. All these calculations with symbolic p give information about the corresponding groups under the Lazard correspondence.

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