1. We will say that an element of a group is *stable* if it is fixed by every automorphism of the group. Describe all finite groups with the property that at least half of the elements are stable.

2. The *centralizer* of a permutation is the set of all permutations that commute with it. What is the minimum number of elements in the centralizer of a permutation in S_n?

3. Prove that the binomial coefficients $\binom{2}{2}, \binom{3}{2}, \binom{4}{2}, \binom{5}{2}, \binom{6}{2}, \ldots$ give all possible remainders modulo n if and only if n is a power of 2.

4. Consider the additive group of polynomials in one variable with rational coefficients of degree at most 5 that take integer values at all integers. This group contains a remarkable subgroup that consists of the polynimials with integer coefficients. Find the index of this subgroup.

5. We are given a table of size 3000×3000 filled with elements of the field $\mathbb{Z}_3$. It is known that the difference of any two columns contains exactly 1000 of 0's, of 1's and of 2's. Prove that the difference of any two rows has the same property.

6. Wedderburn Theorem states that all finite division rings are commutative. We will say that a unital associative ring is an *anti-division ring* if it does not contain invertible elements other than 1. Prove the following “Anti-Wedderburn Theorem”: all finite anti-division rings are commutative.