1. The sum $\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{1200}$ is rewritten as a fraction $\frac{m}{n}$. Prove that m is divisible by 1201.

2. A square matrix will be called *magic* if all row sums, column sums and diagonal sums (i.e., for the main diagonal and for the secondary diagonal) are equal to each other. Find the dimension of the space of all magic $n \times n$ matrices.

3. Let $\mathbb{K}$ be a field.

   a) For any $f, g \in \mathbb{K}[x]$, prove that $f(x)g(y) - f(y)g(x) \in \mathbb{K}[x, y]$ is divisible by $x - y$.

   b) Define $[f, g] = \frac{f(x)g(y)-f(y)g(x)}{x-y}|_{x=y}$. Show that $\mathbb{K}[x]$ with operation $[f, g]$ is a *Lie algebra*, i.e., the antisymmetry identity $[f, f] = 0$ and Jacobi identity $[[f, g], h] + [[g, h], f] + [[h, f], g] = 0$ hold.

   c) Prove that if $\mathbb{K}$ has characteristic zero, then the Lie algebra $L = \mathbb{K}[x]$ defined in (b) is *simple*, i.e., for any ideal I of L (a subspace $I \subset L$ with $[L, I] \subset I$), we have either $I = 0$ or $I = L$.

4. A finite abelian group will be called *balanced* if the sum of its elements is equal to zero. Which is greater: the number of balanced groups or unbalanced groups of order ≤ 2009?

5. a) Help Professor A. B. Normal to prove the following important result: Theorem 3. *If a finite group contains exactly 3 non-normal subgroups, then its order is divisible by 3.*

   b) Can one replace the 3's in this theorem by 4's?

   c) The groups S_3 and D_4 have, respectively, 3 and 4 non-normal subgroups. Does there exist a finite group with exactly 2 non-normal subgroups?

6. Show that for real matrices A the following implication holds:

$$A^{2008} = A^T \quad \Longrightarrow \quad A^{2010} = A.$$