

1. Does the group S_n have more elements of even order or of odd order?
2. Let F be a field, $M_n(F)$ the space of $n \times n$ matrices over F , and $T : M_n(F) \rightarrow M_n(F)$ a linear map such that $\det(A) = \det(T(A))$ for all $A \in M_n(F)$. Prove that T is an invertible map.
3. A student calls a square real matrix A *practically invertible* if there exists a matrix B such that the entries of $C = AB$ differ from the corresponding entries of the identity matrix by no more than 10^{-10} :

$$|c_{ij} - \delta_{ij}| \leq \frac{1}{10000000000} \text{ for all } i, j.$$

Do there exist non-invertible matrices that are practically invertible?

4. Find the positive integers n such that there exists a group of order n that has exactly four (nontrivial) Sylow subgroups.