Lie semisimple algebras of derivations and varieties of almost polynomial growth

Sebastiano Argenti

In this talk we are interested in the growth of the differential identities of finite dimensional associative algebras, i.e., polynomial identities of algebras with an action of a Lie algebra by derivations.

In this context Gordienko and Kochetov proved that, over fields of characteristic zero, the exponent always exists and is a positive integer. As a consequence of their proof, the codimension sequence associated to the differential identities either grows polynomially or exponentially [1].

We are interested in characterizing associative algebras A of almost polynomial growth, that is the codimension sequence of A grows exponentially but for any finite dimensional algebra in the variety generated by A, generating a proper subvariety, the corresponding growth is only polynomial.

This problem appears naturally in the study of varieties of PI-algebras with (or without) some additional structure and was solved for associative algebras [2], graded algebras [3], algebras with involution [4], trace algebras [5], etc...

A common flavor of these results is the existence of a finite number of such varieties and the indication of a concrete list of generating algebras. Our main result concerns the characterization of varieties of almost polynomial growth in case the Lie algebra acting by derivations is $\mathfrak{sl}_2$. In this case we obtain two infinite classes of algebras generating distinct differential varieties of almost polynomial growth.

We also discuss the more general case in which L is any finite dimensional semisimple Lie algebra. We exhibit a list of algebras such that the codimension growth of a variety is exponential if and only if it contains one of the algebras in the list.

References

[1] A. S. Gordienko, M. V. Kochetov; Derivations, gradings, actions of algebraic groups, and codimension growth of polynomial identities. Algebras and Representation Theory, 17(2):539-563, mar 2014.

[2] A. R. Kemer; Nonmatrix varieties. (Russian) Algebra i Logika 19 (1980), no. 3, 255283, 382.

[3] A. Valenti; Group graded algebras and almost polynomial growth. J. Algebra 334 (2011), 247254.

[4] A. Giambruno, S. Mishchenko; On star-varieties with almost polynomial growth. Algebra Colloq. 8 (2001), no. 1, 3342.

[5] A. Ioppolo, P. Koshlukov, D. La Mattina; Trace identities and almost polynomial growth. J. Pure Appl. Algebra 225 (2021), no. 2, Paper No. 106501, 20 pp