

AAC Colloquium Talk

Hurwitz groups: The classification problem

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Abstract

A nontrivial finite group H is called Hurwitz if it is generated by two elements X, Y satisfying the conditions $X^2 = Y^3 = (XY)^7 = 1$. Hurwitz groups come from topology: the automorphism groups of certain compact Riemann surfaces are Hurwitz. The property of a group to be Hurwitz is inherited by quotient groups. Therefore, the classification of simple Hurwitz groups is a problem of primary interest. (Abelian groups are not Hurwitz, so every Hurwitz group is perfect.) Although the problem attracts attention from the beginning of 20-th century, it remains very difficult although all finite simple groups are known. There is no understanding why certain groups are Hurwitz. At the moment there are two approaches to the study of simple and quasi-simple Hurwitz groups. One is of combinatorial nature which allows one to prove that all in a certain sense large finite simple groups are Hurwitz. For instance, the alternating groups A_n are Hurwitz for $n > 167$, and all simple groups $PSL(n; q)$ are known to be Hurwitz for $n > 251$. The second approach is based on the representation theory. It is very efficient for proving that many classical simple groups in relatively small dimension n are not Hurwitz, and in some special cases the method allows one to prove the opposite. More details will be provided in the talk. The bibliography below includes few papers which contain the most recent results and historical comments.