

Coalgebras and Bilinear Forms

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Abstract

A coalgebra over a field k is a triple (C, Δ, ϵ) where C is a k -vector space, the comultiplication Δ maps C to $C \otimes C$, and the counit ϵ maps C to k . The concept is dual to that of a k -algebra, and if the vector space is finite dimensional, then k -coalgebras are exactly the duals of k -algebras. An example of a k -coalgebra is the n^2 dimensional space $M^c(n, k)$ with basis e_{ij} , $1 \leq i, j \leq n$. On basis elements, the comultiplication Δ is given by $\Delta(e_{ij}) = \sum_{k=1}^n e_{ik} \otimes e_{kj}$ and the counit ϵ is given by $\epsilon(e_{ij}) = \delta_{ij}$. This coalgebra is the dual of the algebra of $n \times n$ matrices over k .

In this talk, I will describe some balanced bilinear forms on matrix and matrix-like coalgebras over a field and show how some simple computations can be used to simplify some existing results. This will include work by an NSERC USRA holder, Randy Rose, over last summer.

In 1975, Sweedler introduced formally the definition of a coring, the generalization of coalgebras but over noncommutative rings. Recently, this topic has been the focus of a great deal of research. At the end of this talk, I will define corings, give some examples and show that balanced bilinear forms again play a crucial role in this theory.