Lie algebras and their generalizations

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The notion of Lie algebra is one of the fundamental concepts of modern mathematics and mathematical physics. In principle, Lie Theory reduced problems on Lie groups, which are of analytic nature, to algebraic problems on Lie algebras, an this led to a study of Lie algebras per se. The first fundamental contributions are due to Wilhelm Killing. In 1888 he obtained the classification of simple finite-dimensional complex Lie algebras: four infinite series (the classical Lie algebras) $\mathbf{A}_n$, $\mathbf{B}_n$, $\mathbf{C}_n$, $\mathbf{D}_n$, and only finitely many other Lie algebras, now called the exceptional Lie algebras and denoted $\mathbf{E}_6$, $\mathbf{E}_7$, $\mathbf{E}_8$, $\mathbf{F}_4$, $\mathbf{G}_2$, of respective dimensions 78, 133, 248, 52, 14.

The invention of quantum groups is one of the outstanding achievements of mathematical physics and mathematics in the late twentieth century. A major event was the discovery by V. Drinfeld and M. Jimbo around 1985 of a class of Hopf algebras which can be considered as one-parameter deformations of universal enveloping algebras of simple finite-dimensional complex Lie algebras. In the talk we are going to discuss the concept of Lie algebra and some modern generalizations of it, including different approaches to the definition of quantum Lie algebras.