Atlantic Algebra Centre

Colloquium Talk

Dr Hugh Thomas

Department of Mathematics and Statistics

University of New Brunswick - Fredericton

Wednesday, January 12, 2011
12:00 p.m.
HH-3017

Garside structures for braid groups

ABSTRACT

As is well-known, the symmetric group S_n can be generated by the simple transpositions s_i = (i i+1). These satisfy the relations that $s_i^2=e$ (the *idempotence relations*), and certain additional *braid relations*:

$$s_is_j=s_js_i \text{ if } |i-j|>1, \; s_is_{i+1}s_i=s_{i+1}s_is_{i+1}.$$

If we consider a group with n-1 generators s_i which satisfy just the braid relations, but not the idempotence relation, we get the braid group on n strands, B_n.

In order to facilitate doing calculations in B_n, it's useful to have a normal form for the elements of the group. This, and more, is provided by what is called the *Garside structure* for the group, the idea of which I shall explain. The first such structure for B_n was found by Garside, but I will focus on a different one, due to Birman-Ko-Lee. This structure generalizes to Artin groups associated to finite Weyl groups, as was shown by Bessis. (In fact, everything I am talking about generalizes to that setting, including Garside's original Garside structure but for simplicity I will for the most part stick to the symmetric group/braid group setting.)

I will give a different description of the Birman-Ko-Lee Garside structure, due to Colin Ingalls and me, in terms of *quiver representations*. In order to do so, I will define quiver representations, and provide some background about them. In particular, the notion of *exceptional sequence*, originally studied for vector bundles by Rudakov and his school, and transported into the setting of quiver representations by Crawley-Boevey and Ringel, will be essential.

Finally, I will explain a bit about ongoing efforts to extend the Birman-Ko-Lee-Bessis Garside structure to braid groups corresponding to other (not necessarily finite) reflection groups.

ABOUT THE SPEAKER

Dr Hugh Thomas is an Associate Professor at UNB – Fredericton. He received his Ph. D. and M. Sc. degrees in Mathematics from the University of Chicago, and his B. Sc. Degree from the University of Toronto. His extensive list of publications, conference and seminar talks includes titles in the areas of Algebraic Combinatorics, Representation Theory of Algebras, Algebraic Geometry.