Zero-divisor Graphs of Group Rings

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Abstract

Let R be a commutative ring with $1 \neq 0$, G be a nontrivial finite group and let $Z(R)$ be the set of zero-divisors of R. The zero-divisor graph of R is defined as the graph $\Gamma(R)$ with the vertex set $Z(R)^* = Z(R) \setminus \{(0)\}$ and two distinct vertices a and b are adjacent if and only if $ab = 0$. In this talk, we investigate the interplay between the ring-theoretic properties of group ring RG and the graph-theoretic properties of $\Gamma(RG)$. We first characterize finite abelian group algebras KG with $diam(\Gamma(KG)) \leq 2$ as well as Artinian commutative group rings RG with $gr(\Gamma(RG) \geq 4$. We also investigate the isomorphic problem for zero-divisor graphs of group rings. It is shown that two finite semisimple group rings are isomorphic if and only if their zero-divisor graphs are isomorphic. It is also shown that rank and cardinality of a finite abelian p-group is completely determined by the zero-divisor graph of its modular group ring, extending a result of Akbari et al (J. Algebra 2004). Finally, we show that a finite noncommutative reversible group ring is completely determined by its zero-divisor graph.