Seminar Talk
Braided version of Shirshov-Witt theorem

V.K. Kharchenko,

FES-Cuautitlán, UNAM, Primero de mayo s/n, Campo 1, CIT,

Cuautitlán Izcalli, Estado de México, 54768, MEXICO,

E-mail: vlad@servidor.unam.mx

The Shirshov–Witt theorem claims that every subalgebra of a free Lie algebra is free. Due to Costant-Milnor-Moore theorem in characteristic zero this theorem can be restated in terms of a free associative algebra: Every Hopf subalgebra of the free algebra $\mathbf{k}\langle x_i\rangle$ (with the coproduct $\delta(x_i) = x_i \otimes 1 +1 \otimes x_i$) is free, and is freely generated by primitive elements. Our aim is to extend this result to free algebras with a braided coproduct as far as possible. By means of P.M. Cohn theory we show that if a subalgebra is a right categorical right coideal, then it is free.

However, a categorical Hopf subalgebra does not always have primitive free generators. For this reason there arises the primitive generation problem for subalgebras, biideals and homomorphic images of connected braided Hopf algebras. We propose a notion of *cosymmetric* coalgebra that naturally generalizes the notion of cocommutative coalgebra, and show that a connected braided Hopf algebra is cosymmetric if and only if it is strictly generated by its primitive elements. We provide sufficient conditions for a subalgebra, biideal, or homomorphic image to be cosymmetric.

If the braiding is involutive, $\tau^2 = \mathrm{id}$, then the braided version of Costant-Milnor-Moore theorem is valid. In this case every braided Hopf subalgebra of $\mathbf{k}\langle x_i\rangle$ indeed is generated by primitive elements.

Colloquium Talk

Lie algebras and their generalizations

The notion of Lie algebra is one of the fundamental concepts of modern mathematics and mathematical physics. In principle, Lie Theory reduced problems on Lie groups, which are of analytic nature, to algebraic problems on Lie algebras, an this led to a study of Lie algebras per se. The first fundamental contributions are due to Wilhelm Killing. In 1888 he obtained the classification of simple finite-dimensional complex Lie algebras: four infinite series (the classical Lie algebras) $\mathbf{A}_n$, $\mathbf{B}_n$, $\mathbf{C}_n$, $\mathbf{D}_n$, and only finitely many other Lie algebras, now called the exceptional Lie algebras and denoted $\mathbf{E}_6$, $\mathbf{E}_7$, $\mathbf{E}_8$, $\mathbf{F}_4$, $\mathbf{G}_2$, of respective dimensions 78, 133, 248, 52, 14.

The invention of quantum groups is one of the outstanding achievements of mathematical physics and mathematics in the late twentieth century. A major event was the discovery by V. Drinfeld and M. Jimbo around 1985 of a class of Hopf algebras which can be considered as one-parameter deformations of universal enveloping algebras of simple finite-dimensional complex Lie algebras. In the talk we are going to discuss the concept of Lie algebra and some modern generalizations of it, including different approaches to the definition of quantum Lie algebras.