Algebra Seminar

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1:00pm - 2:00pm,
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Twisting Spin, Half-spin, and Hopf Algebras

Abstract

It is well known that central simple algebras are twisted forms of matrix algebras. That is, when we consider a central simple algebra A over a field F, if we extend scalars to the separable closure of F we have an isomorphism between A_{sep} and $M_n(F_{sep})$ for some n. Conversely, by modifying the natural Galois action on $M_n(F_{sep})$ in a process called twisting, we can identify A as a set of fixed points within $M_n(F_{sep})$. This procedure readily extends to linear algebraic groups which come from subgroups of $M_n(F)$, such as SL_n or SO_n, to produce their twisted counterparts, $SL(A)$ or $SO(A,t)$, where t is an orthogonal involution on A. This twisting result applies to all classical linear algebraic groups, in particular we show that we can twist $Spin_n$ and $HSpin_n$ to produce $Spin(A,t)$ and $HSpin(A,t)$. In addition, and in order to produce the result about $HSpin$, we discuss how the twisting story for linear algebraic groups is mirrored in their representing Hopf algebras. If H is the Hopf algebra of a group G, and $G(A)$ is the twisted form, we show how to construct a modified Galois action on H_{sep} whose fixed points are the representing Hopf algebra of $G(A)$.