# Minimal varieties of graded algebras

Sebastiano Argenti
*Memorial University of Newfoundland, St. John's*

1st October 2025

A cornerstone result in the theory of polynomial identities is the existence of the exponent of any variety of associative algebras, proved by Giambruno and Zaicev at end of the last century. Their theorem allows to classify the varieties on an integral scale whose steps are the minimal varieties of given exponent $d$, namely those varieties of exponent $d$ such that every proper subvariety has exponent strictly less than $d$. Some years later, Giambruno and Zaicev gave also a characterization of the minimal varieties: they are precisely those with factorable $T$-ideal of identities. Moreover, they can also be characterized by being precisely those generated by the Grassmann envelope of "minimal algebras", that in finite dimension are equivalent to upper block triangular matrix algebras.

In this talk we give an account of the efforts made to generalize the previous results to graded algebras and graded polynomial identities. The existence of the graded exponent was proved by Aljadeff and Giambruno. On the other hand, the classification of the minimal varieties of graded algebras poses several challenges. It was found that minimal graded algebras do not produce minimal varieties. Moreover, the graded $T_G$-ideal of minimal varieties is not always factorable. Therefore, new techniques are needed to deal with the graded case. When the grading group $G$ is finite abelian, both the classification problem and the factorability problem were recently solved.