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AAC Mini Course
Introduction to Affine Group Schemes

Affine group schemes are an important generalization of algebraic matrix groups. They arise naturally, for example, when one studies homomorphisms of algebraic matrix groups in characteristic $p > 0$. It is well-known that such a homomorphism can be bijective, but still fail to be an isomorphism. A classic example is the Frobenius homomorphism: it is bijective (over perfect fields), but the inverse is not a polynomial map.

It turns out that one can define kernels in such a way that the Frobenius homomorphism will have a nontrivial kernel, although this kernel will not be an algebraic group - it will be an algebraic group scheme! Working in the category of group schemes, one recovers the usual isomorphism theorems of group theory, which are missing for algebraic groups in characteristic p. One also recovers a nice duality theory for abelian group schemes (the so-called Cartier duality).

In this course we will introduce the concept of affine group scheme, look at the main examples, and develop the basic tools such as the decomposition into irreducible components, dimension, tangent Lie algebra, and the theory of quotients.

At the end, we will look at some applications of affine group schemes to the study of gradings by abelian groups on algebras over a field of prime characteristic.

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Four lectures have been scheduled: November 12, 19, 26 and December 3, 2008. Everyone is invited! A limited support is available for the mathematics students in Atlantic Canada.