Memorial University of Newfoundland
Atlantic Association for Research in the Mathematical Sciences
Atlantic Algebra Centre
CRG "Groups, Rings, Lie and Hopf algebras"

Automorphisms and derivations in affine algebraic geometry

Mini-course

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Leonid Makar-Limanov received his Doctorate degree from Moscow State University in 1970. His Ph D Thesis included now classical description of the automorphisms of free algebras of rank 2. In 1981 he emigrated to the United States and began his work as Professor at Wayne State University. He researches in a number of areas of Mathematics, with a stress on Algebraic geometry, Associative rings and algebras and Nonassociative rings and algebras. He has been a popular lecturer and gave lectures and mini courses in a number of countries including Kazakhstan, Russia, India, Brazil and China. As a researcher, he visited many countries, to name a few, United Kingdom, Spain, Israel, Germany, and Brazil. For his achievements in Mathematics, in 2013 he was named an Inaugural Fellow of the American Mathematical Society

Brief description of the mini course

After this course you will learn the proofs of several classical theorems of Affine Algebraic Geometry. The original proofs of these theorems were quite involved, and a much longer course would be needed for their exposition.

In the first lecture we will discuss the theorems of Heinrich Jung and Rudolf Rentschler. The first one describes all invertible transformations of the plane by polynomials and the second all generalized shifts of the plane. Algebraically speaking, Jung's theorem describes all automorphisms of the ring of polynomials with two variables and Rentschler theorem describes all subgroups of this group which are isomorphic to the group of complex numbers under addition. If we have time, we will discuss the groups of polynomial automorphisms of several other surfaces.

The second lecture is devoted to the following topic: if a cylinder is given, is it possible to recover the base of this cylinder. In general the answer is no, but we discuss two cases when this is possible. We show that if the cylinder over a curve is given then we can recover this curve (this is the theorem of Shreeram Abhyankar, Paul Eakin, and William Heinzer). If the cylinder over a surface is isomorphic to a three-dimensional space then the surface is isomorphic to a plane (this is a theorem of Takao Fujita).

Here is an algebraic translation:

If A is an integral domain of transcendence degree one and $A[x_1, x_2,\dots, x_n]$ is given, we can recover A up to an isomorphism. If A is an integral domain of transcendence degree two and $A[x]$ is isomorphic to $\mathbf{C}[y_1,y_2,y_3]$ then A is isomorphic to $\mathbf{C}[z_1,z_2]$. The main tool used in these two lectures is locally nilpotent derivations.

In the third lecture we prove one of the most famous theorems in affine algebraic geometry, the AMS Theorem (after Abhyankar, Tsuong-tsieng Moh, Masakazu Suzuki): any smooth "good" embedding of a line to a plane is the image of a coordinate line under an automorphism of the plane. Algebraically, this means the following: if two polynomials $f(t), g(t) \in \mathbf{C}[t]$ generate $\mathbf{C}[t]$ then the smaller of the degrees of $f(t), g(t)$ divides the larger of the degrees of $f(t), g(t)$. The main tool here is a new algorithm for finding an irreducible dependence between two polynomials in one variable.

The lectures will be delivered during three time periods, as shown below. They will take place at the St. John's campus of Memorial University and will be broadcast via Webex. All the times are in Newfoundland Time (NST=UTC-3:30).

Monday, March 13th: 3:30 to 5 pm in A-2065

Wednesday, March 15th: 3:30 to 5 pm in A-2065

Thursday, March 16th: 11 am to 12:30 pm in ED-4015

The lectures will be available online via Webex. The details can be found at https://www.mun.ca/aac/mini-courses-2016--/leonid-makar-limanov/ .