ON STRUCTURE OF FILIFORM LIE ALGEBRAS

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The classification of nilpotent Lie algebras is a difficult problem widely discussed in literature. Nilpotent Lie algebras up to dimension 7 are well-known. In 60s M. Vergne introduced the concept of a *filiform* Lie algebra. This is a nilpotent Lie algebra which nil-index is $n-1$ for a given dimension n. An infinite-dimensional analog of a filiform Lie algebra is a so-called Lie algebra of *maximal class* (or of coclass 1). Vergne has shown that an arbitrary filiform Lie algebra is isomorphic to some deformation of the graded filiform Lie algebra $\mathrm{m}_0(n)$ defined by its basis $e_1, \ldots, e_n$ and nontrivial Lie products: $[e_1, e_i] = e_{i+1}$, $i = 2, \ldots, n-1$. In the case of infinite-dimensional $\mathbb{N}$-graded Lie algebras g of maximal class, Vergne proved that if g is generated by the first graded component then it must be isomorphic to m_0 (the direct limit of $\mathrm{m}_0(n)$). The classification of $\mathbb{N}$-graded Lie algebras of maximal class $L = \oplus_{i\in\mathbb{N}} L_i$ generated by L_1, L_2 was originally obtained by A. Fialowski. In line with Vergne's result we proved the following: if g is an $\mathbb{N}$-graded Lie algebra of maximal class generated by graded components of degrees 1 and q, respectively, then under some technical condition there can only be one isomorphism type.

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