

Atlantic Association for Research in Mathematical Sciences  
Memorial University of Newfoundland  
Atlantic Algebra Centre

VI Annual Meeting

# COMBINATORIAL ALGEBRA MEETS ALGEBRAIC COMBINATORICS

*January 16 - January 18, 2009*

## Schedule

## Abstracts of Talks

## Rooms

*All talks will be delivered in Room SN-3060 (Science Building).*

*Coffee breaks/discussions will be in Coffee Room of the Department of Mathematics and Statistics HH-3022 (Henrietta Harvey Building).*

*You are always welcome to visit the office of Atlantic Algebra Centre at HH-2010.*

FRIDAY, JANUARY 16, 2009

- 2:30 - 2:50 HH-3022 **Registration/Opening of the meeting**  
3:00 - 3:50 SN-3060 **Hugh Thomas**  
*University of New Brunswick, Fredericton*  
CLUSTER CATEGORIES, HIGHER CLUSTER CATEGORIES,  
AND NEGATIVE CLUSTER CATEGORIES,  
FROM A COMBINATORIAL PERSPECTIVE
- 3:50 - 4:20 HH-3022 *Coffee/discussions*
- 4:20 - 5:10 SN-3060 **Jaydeep Chipalkatti**  
*University of Manitoba*  
ON THE SATURATION SEQUENCE OF THE RATIONAL  
NORMAL CURVE

SATURDAY, JANUARY 17, 2009

- 9:30 - 10:20 SN-3060 **Francois Bergeron**  
*Université du Québec à Montréal*  
MULTIVARIATE DIAGONAL HARMONIC POLYNOMIALS
- 10:20 - 10:50 HH-3022 *Coffee/discussions*
- 10:50 - 11:40 SN-3060 **Marcelo Aguiar**  
*Texas A&M University, USA*  
THE HOPF MONOID  
OF GENERALIZED PERMUTAHEDRA
- 11:50 - 12:40 SN-3060 **Mitja Mastnak**  
*Saint Mary's University*  
COMBINATORIAL HOPF ALGEBRAS IN FREE PROBABILITY
- L U N C H
- 2:30 - 3:20 SN-3060 **Olga Kharlampovich**  
*McGill University*  
EQUATIONS AND ALGORITHMIC PROBLEMS  
IN FREE AND RESIDUALLY FREE GROUPS
- 3:20 - 3:50 HH-3022 *Coffee/discussions*
- 3:50 - 4:40 SN-3060 **Jerzy Weyman**  
*Northeastern University, USA*  
TOWARDS EQUIVARIANT BOIJ-SODERBERG CONJECTURES

SUNDAY, JANUARY 18, 2009

9:30 - 10:20 SN-3060 **Matej Brešar**  
*University of Ljubljana*  
*and University of Maribor, Slovenia*  
ON VALUES OF NONCOMMUTATIVE POLYNOMIALS  
AND LIE IDEALS

10:20 - 10:40 HH-3022 *Coffee/discussions*

10:40 -11:30 SN-3060 **Christopher Brav**  
*University of Toronto*  
THE PROJECTIVE MCKAY CORRESPONDENCE

**Closure of the meeting**

**Marcelo Aguiar**

*Texas A&M University, USA*

THE HOPF MONOID OF GENERALIZED PERMUTAHEDRA

Joyal's notion of species constitutes a good framework for the study of certain algebraic structures associated to combinatorial objects. We discuss the notion of "Hopf monoid" in the category of species and illustrate it with several examples. We introduce the Hopf monoid of generalized permutahedra (the latter are certain polytopes recently studied by Postnikov, Reiner and Williams). Our main result is an explicit antipode formula for this Hopf monoid. We explain how reciprocity theorems of Stanley on graphs and of Billera, Jia and Reiner on matroids can be deduced from this result.

The talk borrows from joint works with Swapneel Mahajan and with Federico Ardila.

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**Francois Bergeron**

*Université du Québec à Montréal*

MULTIVARIATE DIAGONAL HARMONIC POLYNOMIALS

The aim of this talk is to present beautiful aspects of the extension of the notion of diagonal harmonic polynomials to the context of polynomials in many sets of  $n$  variables. This extension opens the door to many new and exciting conjectures replete with both combinatorics and representation theoretic questions. Hopefully, nice geometrical question will also arise.

# Christopher Brav

*University of Toronto*

## THE PROJECTIVE MCKAY CORRESPONDENCE

Alexander Kirillov Jr. has described a McKay correspondence for finite subgroups of  $PSL_2(\mathbb{C})$  which associates to each ‘height’ function an affine Dynkin quiver, together with a derived equivalence between equivariant sheaves on  $\mathbb{P}^1$  and representations of this quiver. The equivalences for various height functions are related by reflection functors for quiver representations.

We develop an analogous story for the cotangent bundle of  $\mathbb{P}^1$ , in which each height function gives a derived equivalence between equivariant sheaves on the cotangent bundle and modules over the preprojective algebra of an affine Dynkin quiver. These various equivalences are related by the spherical twists of Seidel-Thomas, which take the place of the reflection functors for  $\mathbb{P}^1$ .

Since there is a well-known equivalence between equivariant sheaves on  $\mathbb{C}^2$  and modules over the preprojective algebra, our construction also provides a bridge between Kirillov’s correspondence for  $\mathbb{P}^1$  and the usual McKay correspondence for  $\mathbb{C}^2$ .

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# Matej Brešar

*University of Ljubljana and University of Maribor, Slovenia*

## ON VALUES OF NONCOMMUTATIVE POLYNOMIALS AND LIE IDEALS

We will consider two different, but related topics in which we have been recently involved (with different coauthors). The first topic was motivated by the recent paper [3] in which the authors found an algebraic version of Connes’ embedding conjecture that concerns the values of noncommutative polynomials on matrix algebras. Based on this, in [2] we considered the following problem: Given an algebra  $A$  and a noncommutative polynomial  $f(X_1, \dots, X_n)$ , describe the linear span  $L$  of all values  $f(a_1, \dots, a_n)$  with  $a_i \in A$ . The fundamental observation is that under very mild restrictions this set is a Lie ideal of  $A$  (meaning that  $[L, A] \subseteq L$ ), and using this the problem can be solved in some algebras. This yields the connection to our second topic, that is, the characterization of Lie ideals through ordinary ideals. This problem was studied thoroughly in [1].

## References

- [1] M. Brešar, E. Kissin, V. Shulman, Lie ideals: from pure algebra to  $C^*$ -algebras, *J. Reine Angew. Math.* **623** (2008), 73-121.
- [2] M. Brešar, I. Klep, Values of noncommutative polynomials, Lie skew-ideals and tracial Nullstellensätze, preprint.
- [3] I. Klep, M. Schweighofer, Connes' embedding conjecture and sums of hermitian squares, *Adv. Math.* **217** (2008) 1816-1837.

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## Jaydeep Chipalkatti

*University of Manitoba*

### ON THE SATURATION SEQUENCE OF THE RATIONAL NORMAL CURVE

Let  $C \subseteq \mathbf{P}^d$  denote the rational normal curve of degree  $d$ , which is defined to be the  $d$ -fold Veronese imbedding of  $\mathbf{P}^1$ . Its defining ideal  $I_C$  in the polynomial ring  $\mathbf{C}[a_0, a_1, \dots, a_d]$  is a well-known classical example of a determinantal ideal. It is a little less well-known that  $I_C$  admits an  $SL_2$ -invariant filtration induced by the quadratic covariants of a generic binary  $d$ -ic.

In this talk I will explain the relevant ideas in classical invariant theory which underlie this filtration, and the notion of the saturation sequence which is the principal source of open problems.

I will outline a theorem about the tail of this filtration which can be demonstrated using Gordan's syzygies.

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## Olga Kharlampovich

*McGill University*

### EQUATIONS AND ALGORITHMIC PROBLEMS IN FREE AND RESIDUALLY FREE GROUPS

I will discuss algebraic (or Diophantine) geometry for free and fully residually free groups. I will also describe some effective combinatorial methods, algorithms (related to infinite words), and open problems that arose in our solution (with A. Myasnikov) of the Tarski's problems on elementary theories of free groups.

## Mitja Mastnak

*Saint Mary's University*

### COMBINATORIAL HOPF ALGEBRAS IN FREE PROBABILITY

I will explain how combinatorial Hopf algebras can be used to study joint distributions of  $k$ -tuples in a noncommutative probability space. In recent joint work with A. Nica we have constructed a Hopf algebra whose multiplication of characters corresponds to free multiplicative convolution of joint distributions. In the case  $k = 1$  our combinatorial Hopf algebra is the well known Hopf algebra of symmetric functions. In this case several classical notions in free probability, such as the  $S$ -transform of Voiculescu, its reciprocal  $1/S$ , and its logarithm  $\log S$ , relate in a natural sense to the sequences of complete, elementary and power sum symmetric functions.

## Hugh Thomas

*University of New Brunswick, Fredericton*

### CLUSTER CATEGORIES, HIGHER CLUSTER CATEGORIES, AND NEGATIVE CLUSTER CATEGORIES, FROM A COMBINATORIAL PERSPECTIVE.

I will present some results on cluster categories from the perspective of Coxeter group combinatorics. (The Coxeter groups in question need not be finite but I will focus on the finite case.) In particular, this includes a bijection between  $m$ -clusters and  $m$ -noncrossing partitions; such bijections have previously been found only in the case that  $m = 1$ . It also includes a proposed definition of the “clusters” in negative cluster categories. This is joint work with Aslak Bakke Buan and Idun Reiten.

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## Jerzy Weyman

*Northeastern University, USA*

### TOWARDS EQUIVARIANT BOIJ-SODERBERG CONJECTURES

In this talk I will describe some joint results with David Eisenbud and Steven Sam on the possible equivariant version of the Boij-Soderberg conjectures on the Betti tables of equivariant modules. I will also discuss the original conjectures, their variants and their proof due to Eisenbud and Schreyer.