Ohm’s Law and Equivalent Resistance

\[ V = IR \]

Georg Simon Ohm (1789-1854)
Ohm’s Law

Electric current, $I$, is a measure of the flow of charge. It is rate of charge with time across a given point in a conductor and is measured in amperes ($A$).

In order to have a current, a potential difference or voltage, $V$, must exist across the wire. It is measured in volts ($V$) and is the electric potential energy per unit charge.

The relationship between current and voltage is defined by the object’s resistance, $R$. It is a measure of the resistance to the motion of charged particles and is measured in ohms ($\Omega$). The current that flows through an object is related to the potential difference. This relationship is the subject of Ohm’s law.

Ohm’s law states that for a conductor with resistance $R$ carrying a current $I$, the potential difference across the resistor is

$$V = IR$$
A resistor (-visible-) is any object that passes current (a conductor) and has some resistance $R$.

Below is a picture of the resistors used in today’s experiment. As current from the power supply flows through each resistor supplies a prescribed amount of resistance, which affects the voltage according to Ohm’s Law.

The coloured bands identify each resistor’s value in Ohms ($\Omega$).
Resistors in Series and Parallel

Resistors Connected in series - A number of resistors are in series when the current through each resistor is the same. You can recognize two resistors in series by the fact that there is only one path for the current to follow from one resistor to the next.

\[ \begin{align*}
I &\quad R_1 &\quad I \\
& & &
\end{align*} \]

Resistors Connected in parallel - A number of resistors are in parallel if the voltage across each resistor is the same. You can recognize two resistors in parallel by the fact that there are multiple paths for the current to take. In the diagram below the current \( I \) splits into two different currents, passes through the two resistors (larger current through smaller resistor) and then recombines to the original current \( I \).

\[ \begin{align*}
I &\quad I_1 &\quad R_1 &\quad I_2 &\quad R_2 &\quad I \\
& & & & & &
\end{align*} \]
Equivalent Resistance

When multiple resistors are connected in a circuit it is useful to consider the **equivalent resistance**, $R_{eq}$, which is the resistance of a single resistor that could replace the multiple resistors and have the same effect on the circuit i.e. draw the same current from the power supply.

The formula for the equivalent resistance is different for both types of connections:

**Series**

$$R_{eq} = R_1 + R_2 + \cdots$$

**Parallel**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$
The Equipment

Equipment list:

- Power supply
- Resistors
- Multimeter: device that measures multiple electric quantities like voltage, current and resistance.
- Leads - insulated conducting wires.
Multimeter

When you use the multimeter you will be instructed to (see diagram):

1. Place one lead in the Common Port and one in the VΩmA (or VRI) Port.
2. Select the setting for $V$, $I$, or $R$.
3. Attach the leads to the object you are measuring:
   a) in parallel for $V$.
   b) in series for $I$.
   c) in parallel with the power disconnected for $R_{eq}$.
Circuit Schematic Diagram

This is a diagram with the appropriate circuit symbols for the circuit you are about to construct.

- Voltmeter (multimeter set to measure voltage)
- Resistor
- Power supply
- Ammeter (multimeter set to measure current)
Constructing the Circuit

- Plug in the power supply but keep the main switch OFF while assembling the circuit.
- Connect the (+) terminal of the 0-30Vdc output on the power supply to the $V\Omega mA$ terminal on the multimeter.
- Connect the COM terminal of the multimeter to one of terminals of one of the resistors on the wooden box.
- Complete the circuit by connecting the other terminal of the resistor to the (–) terminal of the 0-30Vdc output of the power supply.
- Connect a second multimeter in parallel with the resistor by connecting leads of the multimeter from the $V\Omega mA$ and the COM into the leads connected to the resistor. Parallel means that each end of the resistor also connects to the multimeter.
Setting the Multimeters

- We have just connected the first multimeter in series with $R_1$ and the second one in parallel with $R_1$.
- The first multimeter will measure current (set to 200 $mA$) and the second will read voltage (set to 20 $V$).

Both of the multimeters should have a reading. If not, it is likely that the circuit is not complete. If you have this problem check your circuit thoroughly by going through the previous instructions. If you can’t find the problem check with an instructor.

If you have a negative reading, you should check the instructions to make sure you’ve connected the leads in the correct terminals of the power supply.
Collecting the Data

- Now that the circuit is ready, you can collect a range of $V$ and $I$.
- Record both of the readings on the multimeters in Table 1.
- You will not need to record uncertainties since you will do a plot and get the uncertainties from the fit.

Do not forget to write down the units in the table header for each column.

- Turn on the power supply and start with a voltage near the low end of the scale. Record the voltage and current in Table 1. Adjust the voltage to get new readings of $V$ and $I$.
- Repeat this procedure so that you have at least 8 measurements and record your data.
Plotting Your Data

Launch Graphical Analysis by clicking the icon between the arrows:

- You should see an empty table with graph of **Voltage** versus **Current**.
- Enter the values from **Table 1** into the appropriate columns in the Graphical Analysis.
- Do a linear fit by clicking **Analyze** then **Linear Fit**.
- Display uncertainties for slope and intercept by double clicking on the fit box that appears and choosing “Show uncertainties” option.
- Record your fit parameters in **Table 2**.

Have an instructor check your slope and initial your Lab Worksheet.

- Print the graph by clicking **File → Print**.
Interpret Your Data

QUESTION 1: Interpret your graph by writing down what the fit parameters represent. Show your reasoning.

QUESTION 2: Use the answer to the previous question to write down the unknown resistance with its experimental uncertainty in the form $R \pm \delta R$.

QUESTION 3: Write down the four colors on your resistor. Write down the resistance of $R_1$ and find the experimental uncertainty using the color code. Also, write $R_1$ in the form $R \pm \delta R$.

You may use the color code and example on the next slide to write down resistance of $R_1$ and its experimental uncertainty for the next question.

QUESTION 4: Do your values from the previous two questions agree within experimental uncertainty? Write down the range of values of both.
Resistor Color Code

The unknown resistors have four colored bands on them. They tell you what the resistance of the resistor is. It works as follows, the first two give you the tens and ones digit.

Example:

1. Interpret Colors
   - 1st is red = 2
   - 2nd is yellow = 4
   - Multiplier is green = 100000
   - Tolerance is silver = 10%

2. \( R = 24 \times 100000 = 2.4 \times 10^6 \ \Omega \)

3. \( \delta R = 10\% \times 2.4 \times 10^6 = \pm 2.4 \times 10^5 \ \Omega \)

4. \( R = (2.4 \pm 0.2) \times 10^6 \ \Omega \)

<table>
<thead>
<tr>
<th>Color</th>
<th>1st</th>
<th>2nd</th>
<th>Multiplier</th>
<th>Tolerance</th>
</tr>
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<td>Black</td>
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<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Brown</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
<td>2</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Orange</td>
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<td></td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
<td>4</td>
<td>10000</td>
<td></td>
</tr>
<tr>
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<td>5</td>
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<td>100000</td>
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<tr>
<td>Violet</td>
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<td>7</td>
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<tr>
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<td></td>
<td>0.1</td>
<td>5%</td>
</tr>
<tr>
<td>None</td>
<td></td>
<td></td>
<td></td>
<td>20%</td>
</tr>
</tbody>
</table>
Measuring Resistance

You will now measure the resistance of the two unknown resistors using the multimeter. Disconnect the power supply and all leads from your circuit, leaving the second multimeter with the two leads connected to COM and $V\Omega mA$ ports.

Make sure that your resistors are not connected to the power supply.

- Switch a multimeter to the $20k\Omega$ setting.
- Measure $R_1$ and $R_2$ by placing leads from the COM and $V\Omega mA$ ports into the terminals of each resistor.
- Record the readings in Table 3 and in next two questions.
- Show any workings for uncertainties in the space below Table 3.

The relative uncertainty is given of the multimeter is $\frac{\delta R}{R} = 0.008$.

You must find the absolute uncertainty, $\delta R$. 
Measuring Equivalent Resistance

- Connect the two resistors on the box in **series** using the leads.
- Do not connect to the power supply!
- Measure the equivalent resistance by connecting the multimeter to the open terminals and record the reading in **Table 3**.
- Show any workings for uncertainties in the space below Table 3.

- Connect the two resistors in **parallel** using the leads.
- Measure the equivalent resistance by connecting the multimeter to two terminals of one resistor.
- Record the reading in **Table 4**.
- Show any workings for uncertainties in the space below Table 4.
Calculating Equivalent Resistance

QUESTION 5: In your lab workbook, draw the circuit diagram for the resistors $R_1$ and $R_2$ in series and the diagram for $R_1$ and $R_2$ in parallel. Include the ohmmeter in your diagrams.

QUESTION 6: Calculate the equivalent resistance of the resistors in series from the measured readings of $R_1$ and $R_2$ along with the experimental uncertainty. Also write your answer in the form $R \pm \delta R$.

QUESTION 7: Show that for two resistors in parallel

$$R_{eq} = \frac{R_1 R_2}{(R_1 + R_2)}$$

Note: This is a derivation, not a calculation, so no numbers!
Calculating Equivalent Resistance

QUESTION 8: Calculate the equivalent resistance of the resistors in parallel from the measured readings of \( R_1 \) and \( R_2 \) along with the experimental uncertainty. Also write your answer in the form \( R \pm \delta R \).

QUESTION 9: Comment on whether the equivalent resistances just calculated for resistors in series agree with those measured earlier. Show the range of values associated with the experimental uncertainty.

QUESTION 10: Comment on whether the equivalent resistance calculated for resistors in parallel agree with those measured earlier. Show the range of values associated with the experimental uncertainty.
Conclusions and Summary

QUESTION 11: If we had plotted current vs. voltage, what would we expect for the slope of the graph?

QUESTION 12: Give one source of systematic uncertainty in this experiment. Give one source of random uncertainty in this experiment.

QUESTION 13: Did your results verify Ohm’s Law? Explain briefly.
Wrap it Up!

- Check that you have completed all the tables in your Laboratory Workbook.
- Be sure that you have answered completely all parts to all questions.
- Make sure that you have printed your voltage vs current graph and stapled it into your laboratory workbook.
- Tidy away your work area and log out of your computer.